

*Review of
Elementary
Algebra
Content*



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FRACTIONS

Lowest Terms

Example:
Simplify: $\frac{12}{15} = \frac{4 \cdot \cancel{3}^1}{5 \cdot \cancel{3}_1}$
 $= \frac{4}{5}$

Addition/Subtraction

Rule: To add and subtract fractions you need a **Common Denominator**.

Example: $\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7}$
 $= \frac{5}{7}$

For this example, we already have the common denominator.

Example: $\frac{1}{4} + \frac{2}{3} = \frac{1}{4} \cdot \frac{3}{3} + \frac{2}{3} \cdot \frac{4}{4}$
 $= \frac{3}{12} + \frac{8}{12}$
 $= \frac{11}{12}$

This example, we need to find a common denominator, or a denominator which is **DIVISIBLE** by 3 and 4. One such denominator is 12.

Example: $\frac{7}{12} - \frac{1}{4} = \frac{7}{12} - \frac{1}{4} \cdot \frac{3}{3}$
 $= \frac{7}{12} - \frac{3}{12}$
 $= \frac{4}{12}$
 $= \frac{1}{3}$

Note: Any number over itself is equal to 1. When you multiply a number by 1, you are not changing the number, just renaming it.

$$\frac{4}{4} = 1$$

$$\frac{3}{3} = 1$$

$$\frac{9}{9} = 1$$

FRACTIONS

Multiplication/Division

Rule: Multiplication and division of fractions do not require a common denominator.

Example:
$$\frac{7}{8} \cdot \frac{12}{5} = \frac{7}{\cancel{8}_2} \cdot \frac{1\cancel{2}^3}{5}$$
$$= \frac{7 \cdot 3}{2 \cdot 5}$$
$$= \frac{21}{10}$$

Cancel a common factor of 4

Multiply across

Example:
$$\frac{2}{3} \div \frac{5}{6} = \frac{2}{3} \cdot \frac{6}{5}$$
$$= \frac{2}{\cancel{3}_1} \cdot \frac{\cancel{6}^2}{5}$$
$$= \frac{2 \cdot 2}{1 \cdot 5}$$
$$= \frac{4}{5}$$

To divide by $\frac{5}{6}$, multiply by the reciprocal which is $\frac{6}{5}$

FRACTIONS

Problems

Answers

Perform the following operations.

- | | | |
|-----|-------------------------------|-----------------|
| 1. | $\frac{7}{12} + \frac{5}{12}$ | 1 |
| 2. | $\frac{2}{3} + \frac{1}{5}$ | $\frac{13}{15}$ |
| 3. | $\frac{3}{4} - \frac{1}{3}$ | $\frac{5}{12}$ |
| 4. | $\frac{5}{6} + \frac{5}{12}$ | $\frac{5}{4}$ |
| 5. | $\frac{5}{7} - \frac{1}{6}$ | $\frac{23}{42}$ |
| 6. | $\frac{7}{12} - \frac{1}{2}$ | $\frac{1}{12}$ |
| 7. | $\frac{3}{4} + 2$ | $\frac{11}{4}$ |
| 8. | $5 - \frac{4}{3}$ | $\frac{11}{3}$ |
| 9. | $\frac{16}{3} - \frac{13}{5}$ | $\frac{41}{15}$ |
| 10. | $\frac{16}{3} - \frac{12}{7}$ | $\frac{76}{21}$ |

FRACTIONS

Problems (continued)

Answers

Perform the following operations.

11. $\frac{25}{4} + 6$

$$\frac{49}{4}$$

12. $\frac{1}{2} \cdot \frac{3}{4}$

$$\frac{3}{8}$$

13. $\frac{3}{7} \cdot \frac{2}{5}$

$$\frac{6}{35}$$

14. $2 \cdot \frac{2}{15}$

$$\frac{4}{15}$$

15. $\frac{3}{5} \div \frac{2}{3}$

$$\frac{9}{10}$$

16. $\frac{3}{8} \div 2$

$$\frac{3}{16}$$

17. $\frac{5}{6} \div \frac{3}{14}$

$$\frac{35}{9}$$

18. $\frac{9}{4} \div \frac{5}{3}$

$$\frac{27}{20}$$

INTEGERS

Definition: The absolute value of a number is the distance between 0 and the number on the number line. The symbol for “The absolute value of a” is $|a|$. The absolute value of a number can never be negative.

Examples:

- 1) $|-4| = 4$
- 2) $|3| = 3$

Addition of Signed Numbers:

- Like signs: Add their absolute values and use the common sign.

Examples:

- 1) $-3 + (-2) = -(3 + 2)$
 $= -5$

- 2) $4 + 8 = (4 + 8)$
 $= 12$

- Unlike signs: Subtract their absolute values and use the sign of the number with the larger absolute value.

Examples:

- 1) $-3 + 4 = +(4 - 3)$
 $= 1$

- 2) $2 + (-6) = -(6 - 2)$
 $= -4$

INTEGERS

Subtraction of Signed Numbers

- 1) Change the subtraction symbol to addition.
- 2) Change the sign of the number being subtracted.
- 3) Add the numbers using like signs or unlike signs rules for addition.

Examples: 1) $-3 - 4 = -3 + (-4)$
 $= -7$

2) $6 - (-2) = 6 + (2)$
 $= 8$

3) $-2 - (-4) = -2 + 4$
 $= 2$

Multiplication and Division of Two Signed Numbers:

- **Like signs:** The product or quotient of two numbers with like signs is positive.

Examples: 1) $(-3)(-4) = 12$

2) $\frac{8}{4} = 2$

3) $\frac{-16}{-4} = 4$
 $= 2$

- **Unlike signs:** The product or quotient of two numbers with unlike signs is negative.

Examples: 1) $(-3)(4) = -12$

2) $\frac{-15}{3} = -5$

- Division by 0 is undefined.

Example: $\frac{3}{0}$ is undefined.

INTEGERS

Addition and Subtraction of Signed Numbers

Problems

Answers

Perform the following operations.

- | | | |
|-----|--|----------------|
| 1. | $5 + (-3)$ | 2 |
| 2. | $-6 + (-2)$ | -8 |
| 3. | $7 - 12$ | -5 |
| 4. | $-11 - 4$ | -15 |
| 5. | $6 + [2 + (-13)]$ | -5 |
| 6. | $8 - (-5)$ | 13 |
| 7. | $-3 - (4 - 11)$ | 4 |
| 8. | $-\frac{5}{6} - \frac{1}{2}$ | $-\frac{4}{3}$ |
| 9. | $[(-9) + (-14)] + 12$ | -11 |
| 10. | $-4.4 - 8.6$ | -13 |
| 11. | $2 + (-4 - 8)$ | -10 |
| 12. | $\frac{9}{10} + \left(-\frac{3}{5}\right)$ | $\frac{3}{10}$ |
| 13. | $-8 - [(-4 - 1) - (9 - 2)]$ | 4 |
| 14. | $[-8 + (-3)] + [-7 + (-6)]$ | -24 |
| 15. | $-4 + [(-12 + 1) - (-1 - 9)]$ | -5 |

INTEGERS

Multiplication and Division of Signed Numbers

Problems

Answers

Perform the following operations.

1. $(3)(-4)$

-12

2. $\frac{24}{-6}$

-4

3. $(-10)(-12)$

120

4. $\frac{0}{-2}$

0

5. $\left(-\frac{3}{8}\right)\left(-\frac{10}{9}\right)$

$\frac{5}{12}$

6. $(-9.8) \div (-7)$

1.4

7. $\frac{-30}{2-8}$

5

8. $(-5.1)(.02)$

-.102

9. $\frac{-40}{8-(-2)}$

-4

10. $(-9-1)(-2)-(-6)$

26

ORDER OF OPERATIONS

Please Excuse My Dear Aunt Sally

Grouping Symbols	{	Parentheses ()	Exponents	Multiply	Divide	Add	Subtract
		Brackets []		"as you see it, left to right"		"as you see it, left to right"	
		Square Roots $\sqrt{\quad}$					
		Absolute Value $ \quad $					

Examples:

Note: When one pair of grouping symbols is inside another pair, perform the operations within the innermost pair of grouping symbols first.

$$36 \div (5 - 2)^2 + 6 \cdot 7$$

$$54 \div [3 - (2 \cdot 3)] - 12 + 3$$

$$36 \div (3)^2 + 6 \cdot 7$$

$$54 \div [3 - (6)] - 12 + 3$$

$$36 \div 9 + 6 \cdot 7$$

$$54 \div [-3] - 12 + 3$$

$$4 + 42$$

$$-18 - 12 + 3$$

$$46$$

$$-30 + 3$$

$$-27$$

Examples:

Note: For a problem with a fraction bar, perform the operations in the numerator and denominator separately.

$$\frac{4|9-7|+|-7|}{3^2-2^2} = \frac{4 \cdot 2 + 7}{9-4} = \frac{8+7}{5} = \frac{15}{5} = 3$$

$$\frac{(6-5)^4 - (-21)}{(-9)(-3) - 4^2} = \frac{1^4 - (-21)}{(-9)(-3) - 4^2} = \frac{1 - (-21)}{(-9)(-3) - 16} = \frac{1+21}{27-16} = \frac{22}{11} = 2$$

ORDER OF OPERATIONS

Problems

Answers

Perform the following operations.

1. $5 + 3 \cdot 5 - 2$

18

2. $12 \div 2 \cdot 6 + 1$

37

3. $4 - 3^2 + 6$

1

4. $6 \cdot 2 - 8 \div 2 + 4$

12

5. $-12 - (4 + 3)$

-19

6. $5 - 2(4 \cdot 3) - 5$

-24

7. $9 \cdot 2 \div 6 - 5(2)^2$

-17

8. $\frac{2 - 4}{(-3)^2 + 1}$

$-\frac{1}{5}$

9. $[4 + 2(2 - 5)^2] - 3$

19

10. $\frac{3}{4} - \left(\frac{2}{3}\right)\left(\frac{1}{2}\right)^2$

$\frac{7}{12}$

EXPONENTS

Laws of Exponents

Definition: $a^n = \underbrace{a \cdot a \cdot a \cdot \dots \cdot a}_{n \text{ times}}$

a = the base

n = the exponent

Properties:

1. Product rule: $a^m a^n = a^{m+n}$ Example: $a^2 a^3 = \underbrace{a \cdot a}_{2 \text{ times}} \cdot \underbrace{a \cdot a \cdot a}_{3 \text{ times}} = a^{2+3} = a^5$

2. Quotient rule: $\frac{a^m}{a^n} = a^{m-n}$ Example: $\frac{a^7}{a^5} = \frac{\overbrace{a \cdot a \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a}}^{1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1}}{\underbrace{\cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a}}_{1 \ 1 \ 1 \ 1 \ 1}} = a^{7-5} = a^2$

3. Power rule: $(a^n)^m = a^{nm}$ Example: $(a^3)^2 = a^{3 \cdot 2} = a^6$

Raising a product to a power:

$$(ab)^n = a^n b^n$$

Raising a quotient to a power:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

4. Zero power rule: $a^0 = 1$ Example: $(x)^0 = 1$; $(3x)^0 = 3^0 \cdot x^0 = 1 \cdot 1 = 1$

Examples:

$$(12ab^2)(3a^3b^5c) = 36a^{1+3}b^{2+5}c = 36a^4b^7c$$

$$\left(\frac{2x^2}{3y^3}\right)^4 = \frac{2^4 x^{2 \cdot 4}}{3^4 y^{3 \cdot 4}} = \frac{16x^8}{81y^{12}}$$

$$3x^2y^2(2xy + 5y^2) = (3x^2y^2)(2xy) + (3x^2y^2)(5y^2) = 6x^{2+1}y^{2+1} + 15x^2y^{2+2} = 6x^3y^3 + 15x^2y^4$$

EXPONENTS

Problems

Answers

Perform the following operations and simplify.

1. $(x^2y^5)(x^8y^3)$ $x^{10}y^8$
2. $(5a^6b)(-7a^3b^8)$ $-35a^9b^9$
3. $\frac{x^{10}y^4}{x^6yz}$ $\frac{x^4y^3}{z}$
4. $\frac{10a^3b^7}{-2a^2b^4}$ $-5ab^3$
5. $(ab^4c^2)^3$ $a^3b^{12}c^6$
6. $(-4x^2y^5z^3)^2$ $16x^4y^{10}z^6$
7. $(-m^3n)^3$ $-m^9n^3$
8. $(3n^0x)^2$ $9x^2$
9. $(5x^2y^3)^0(-3xy^4)^3$ $-27x^3y^{12}$
10. $\left(\frac{-4x^3y}{2xyz}\right)^3$ $\frac{-8x^6}{z^3}$

EXPONENTS

Negative Exponents

Definitions:

$$\left. \begin{array}{l} a^{-m} = \frac{a^{-m}}{1} \searrow \frac{1}{a^m} \\ \frac{1}{a^{-m}} \nearrow \frac{a^m}{1} = a^m \\ \frac{a^{-m}}{b^{-n}} \swarrow \frac{b^n}{a^m} \end{array} \right\}$$

Note: The act of moving a factor from the numerator to the denominator (or from the denominator to the numerator) changes a negative exponent to a positive exponent.

Examples:

$$10^{-4} = \frac{10^{-4}}{1} \searrow \frac{1}{10^4} = \frac{1}{10,000}$$

$$ab^{-2} = \frac{ab^{-2}}{1} = \frac{a^1b^{-2}}{1} \searrow \frac{a^1}{b^2} = \frac{a}{b^2}$$

Since b^{-2} has a negative exponent, move it to the denominator to become b^2

$$\frac{x^{-5}y^2}{z^{-3}} = \frac{\boxed{x^{-5}}y^2}{\boxed{z^{-3}}} = \frac{y^2\boxed{z^3}}{\boxed{x^5}}$$

Only the factors with negative exponents are moved.

EXPONENTS

Example:

$$\left(\frac{-12x^{-2}y^3}{3x^4y^2}\right)^{-2} = \left(\frac{-12y^3}{3x^2x^4y^2}\right)^{-2}$$

Within the parenthesis, move the factors with negative exponents.

$$= \left(\frac{-4y}{x^6}\right)^{-2}$$

Simplify within the parenthesis.

$$\left. \begin{aligned} &= \frac{(-4)^{-2}y^{-2}}{(x^6)^{-2}} \\ &= \frac{(-4)^{-2}y^{-2}}{x^{-12}} \end{aligned} \right\}$$

Use the power rule for exponents.

$$= \frac{x^{12}}{(-4)^2y^2}$$

Move all factors with negative exponents.

$$= \frac{x^{12}}{16y^2}$$

Simplify.

EXPONENTS

Problems

Perform the following operations and simplify.

1. $(-3)^2$

9

2. -3^2

-9

3. $3^0 \cdot 3^{-12} \cdot 3^8$

$$\frac{1}{3^4} = \frac{1}{81}$$

4. $\left[\frac{4 \cdot 5^{-1}}{2^{-3}} \right]^{-2}$

$$\frac{5^2}{4^2 2^6} = \frac{25}{1024}$$

5. $(3a^0 b^4)(3ab^3 c^2)$

$$9ab^7 c^2$$

6. $(-3ebc^2)^2 (2ab^{-13})$

$$\frac{18ac^4 e^2}{b^{11}}$$

7. $(x^3 y^{-2})^{-3}$

$$\frac{y^6}{x^9}$$

8. $\frac{6ab^0 c^{-2}}{(-3a)^{-1} bc^{-3}}$

$$\frac{-18a^2 c}{b}$$

EXPONENTS

Problems (continued)

Answers

9.	x^{-8}	$\frac{1}{x^8}$
10.	2^{-3}	$\frac{1}{8}$
11.	$a^{-4} a^{10} a^{-8}$	$\frac{1}{a^2}$
12.	$a^{-6}(a^2 a^{-3})$	$\frac{1}{a^7}$
13.	$\frac{x^8}{x^{-4}}$	x^{12}
14.	$\frac{b^{-3}}{b^5}$	$\frac{1}{b^8}$
15.	$\frac{a^{-4}}{a^{-6}}$	a^2
16.	$\frac{p}{p^{-5}}$	p^6
17.	$\frac{4^{-5}}{4^{-2}}$	$\frac{1}{64}$
18.	$\frac{2^8}{2^5}$	8
19.	$\frac{2^{-1}}{2^2}$	$\frac{1}{8}$
20.	$\frac{ab^{-4}}{a^5 b^{-3}}$	$\frac{1}{a^4 b}$
21.	$\frac{x^{-5} y^2}{x^{-2} y^{-3}}$	$\frac{y^5}{x^3}$

EXPONENTS

Problems (continued)

Answers

- | | | |
|-----|---|---------------------------|
| 22. | $\frac{b^{-2}c^{-8}}{b^4c^{-2}}$ | $\frac{1}{b^6c^6}$ |
| 23. | $\frac{3^{-4}}{9^{-3}}$ | 9 |
| 24. | $\frac{2^{-5}a^{-3}}{2a^{-6}}$ | $\frac{a^3}{64}$ |
| 25. | $(2a^4b^{-3})^5$ | $\frac{32a^{20}}{b^{15}}$ |
| 26. | $(3x^{-4})^{-3}$ | $\frac{x^{12}}{27}$ |
| 27. | $3a^{-4}(2a^{-5}b)^{-2}$ | $\frac{3a^6}{4b^2}$ |
| 28. | $(2a^3b^{-4})^{-1}(a^{-6}b)^{-2}$ | $\frac{a^9b^2}{2}$ |
| 29. | $(-4a^5b^{-2})^{-2}$ | $\frac{b^4}{16a^{10}}$ |
| 30. | $\frac{(a^{-4}b^5)^{-3}}{a^{-7}b^2}$ | $\frac{a^{19}}{b^{17}}$ |
| 31. | $-5a^{-1}(a^3b^{-4})^{-6}$ | $\frac{-5b^{24}}{a^{19}}$ |
| 32. | $\frac{(2a^5b^{-4})^2}{(2a^{-3}b^2)^{-3}}$ | $\frac{32a}{b^2}$ |
| 33. | $\frac{3a^{-2}(2a^{-5})^{-4}}{(3a^2)^{-2}}$ | $\frac{27a^{22}}{16}$ |

POLYNOMIALS

Definition: A sum of a finite number of terms of the form: $a_n x^n$ where a_n is a real number and n is a non-negative integer. (No negative exponents, no fractional exponents.)

Examples: $3x^4 + 2x^3 - 8x + 1$ is a polynomial
 $2x^{-5} + 3x^2$ is not a polynomial

Types of Polynomials:

- Monomial: A polynomial with 1 term

Example: $3x^4$

- Binomials: A polynomial with 2 terms

Example: $2x^3 + 4$

- Trinomial: A polynomial with 3 terms

Example: $3x^2 + 4x - 2$

Degree:

- Degree of a term – the sum of the exponents on the variables.

Example: $3x^4 y^3$ has degree $4 + 3 = 7$

- Degree of a polynomial – the largest degree of any of the terms. In a polynomial with one variable it is the largest exponent.

Example:

$5x^4 - 3x^3 + 2x^2 - 8$ has degree 4

$2x^2 y + 3xy - 4$ has degree 3 since $2x^2 y$ has degree $2 + 1 = 3$

POLYNOMIALS

Addition of Polynomials

Adding Polynomials: Combine like terms.

Example:

$$(3x^2 + 4x - 5) + (2x^2 - 8x + 2) = 3x^2 + 4x - 5 + 2x^2 - 8x + 2 = 5x^2 - 4x - 3$$

Subtraction of Polynomials

Subtracting Polynomials: Distribute the negative sign and combine like terms.

Example:

$$(3x^2 + 4x - 5) - (2x^2 - 8x + 2) = 3x^2 + 4x - 5 - 2x^2 + 8x - 2 = x^2 + 12x - 7$$

Multiplication of Polynomials

Multiplying Polynomials: Use the distributive property.

Example:

$$\begin{aligned}(3x + 2)(x^2 - 3x + 4) &= 3x(x^2 - 3x + 4) + 2(x^2 - 3x + 4) \\ &= 3x^3 - 9x^2 + 12x + 2x^2 - 6x + 8 \\ &= 3x^3 - 7x^2 + 6x + 8\end{aligned}$$

Example:

$$\begin{aligned}(2x + 3)(3x - 4) &= (2x)(3x) + (2x)(-4) + 3(3x) + 3(-4) \\ &= 6x^2 - 8x + 9x - 12 \\ &= 6x^2 + x - 12\end{aligned}$$

Example:

$$\begin{aligned}(3y + 4)(3y - 4) &\text{ The multiplication of the sum and difference of two terms.} \\ &= 3y(3y) + 3y(-4) + 4(3y) + 4(-4) \\ &= 9y^2 - 12y + 12y - 16 \\ &= 9y^2 + \begin{matrix} \searrow & \swarrow \\ & 0 \end{matrix} - 16 \\ &= 9y^2 - 16 \quad \text{The answer is "the difference of two squares."}\end{aligned}$$

POLYNOMIALS

Example:

$(4y + 3)^2$ The square of a binomial.

$$\begin{aligned}(4y + 3)(4y + 3) &= 4y(4y) + 4y(3) + 3(4y) + 3(3) \\ &= 16y^2 + 12y + 12y + 9 \\ &= 16y^2 + 2(12y) + 9 \\ &= 16y^2 + 24y + 9 \quad \text{The answer is a "perfect square trinomial."}\end{aligned}$$

Example:

$(a - b)^3$ The cube of a binomial.

$$\begin{aligned}(a - b)(a - b)(a - b) &= [(a - b)(a - b)](a - b) \\ &= [a \cdot a + a(-b) - b(a) - b(-b)](a - b) \\ &= [a^2 - ab - ab + b^2](a - b) \\ &= [a^2 - 2ab + b^2](a - b) \\ &= a^2(a - b) - 2ab(a - b) + b^2(a - b) \quad \text{Use the distributive property.} \\ &= a^3 - a^2b - 2a^2b + 2ab^2 + ab^2 - b^3 \\ &= a^3 - 3a^2b + 3ab^2 - b^3\end{aligned}$$

Division of Polynomials

Dividing a polynomial by a monomial.

Example:

$$\begin{aligned}\frac{9x^4y^3 - 24x^2y^5 + xy^4}{3x^2y^2} &= \frac{9x^4y^3}{3x^2y^2} - \frac{24x^2y^5}{3x^2y^2} + \frac{xy^4}{3x^2y^2} \\ &= 3x^{4-2}y^{3-2} - 8x^{2-2}y^{5-2} + \frac{1}{3}x^{1-2}y^{4-2} \\ &= 3x^2y - 8x^0y^3 + \frac{1}{3}x^{-1}y^2 \\ &= 3x^2y - 8y^3 + \frac{y^2}{3x}\end{aligned}$$

POLYNOMIALS

Problems

Find the degree of the following and determine what type of polynomial is given.

1. $3x^3y^2 + 2xy - 8$

2. $5 - 2x^3$

3. x^5

4. 8

Add:

5. $(3x^3 + 2x^2 - 5x + 4) + (6x^3 - 2x - 8)$

Subtract:

6. $(3x^3 + 2x^2 - 5x + 4) - (6x^3 - 2x - 8)$

Multiply:

7. $(x + 2)(2x^2 - 5x + 3)$

8. $(3x + 2)(2x - 3)$

9. $(4x + 3)(4x - 3)$

10. $(5x + 7)^2$

11. $(9y - 2)^2$

12. $(3x - 4)^3$

Divide:

13.
$$\frac{12x^{15} - 48x^{12} + x^{10} - 18x^8}{6x^{10}}$$

Answers

Trinomial, degree 5

Binomial, degree 3

Monomial, degree 5

Monomial, degree 0

$$9x^3 + 2x^2 - 7x - 4$$

$$-3x^3 + 2x^2 - 3x + 12$$

$$2x^3 - x^2 - 7x + 6$$

$$6x^2 - 5x - 6$$

$$16x^2 - 9$$

$$25x^2 + 70x + 49$$

$$81y^2 - 36y + 4$$

$$27x^3 - 108x^2 + 144x - 64$$

$$2x^5 - 8x^2 + \frac{1}{6} - \frac{3}{x^2}$$

FACTORING

Factoring Out the Greatest Common Factor

1. Identify the TERMS of the polynomial.
2. Factor each term to its prime factors.
3. Look for common factors in all terms.
4. Factor out the common factor.
5. Check by multiplying.

Example: Factor $6x^2 + 8xy^2 - 4x$

Terms: $6x^2$, $8xy^2$, and $-4x$

$$2 \cdot 3 \cdot x \cdot x + 2 \cdot 2 \cdot 2 \cdot x \cdot y \cdot y - 2 \cdot 2 \cdot x$$

Factor into primes

$$\textcircled{2} \cdot \textcircled{3} \cdot \textcircled{x} \cdot x + \textcircled{2} \cdot 2 \cdot \textcircled{2} \cdot \textcircled{x} \cdot y \cdot y - 2 \cdot \textcircled{2} \cdot \textcircled{x}$$

Look for common factors

Answer: $2x(3x + 4y^2 - 2)$

Factor out the common factor

Check: $2x(3x) + 2x(4y^2) - 2x(2)$

Check by multiplying

$$6x^2 + 8xy^2 - 4x$$

Factoring Trinomials of the Form: $x^2 + bx + c$

To factor $x^2 + 5x + 6$, look for two numbers whose product = 6, and whose sum = 5

List factors of 6	{	Choose the pair which adds to 5 and multiplies to 6.
1•6		$\frac{2}{-2} + \frac{3}{-3} = 5$
-1•-6		$\frac{2}{-2} \cdot \frac{3}{-3} = 6$
2•3	←	
-2•-3		

Since the numbers 2 and 3 work for both, we will use them.

So, $x^2 + 5x + 6 = x^2 + 2x + 3x + 6$

Substitute $2x + 3x$ in for $5x$.

$$= x(x + 2) + 3(x + 2)$$

Factor by grouping.

$$= (x + 2)(x + 3)$$

Answer: $(x + 2)(x + 3)$

Check by multiplying: $(x + 2)(x + 3) = x^2 + 2x + 3x + 6$
 $= x^2 + 5x + 6$

FACTORING

Factoring Trinomials of the Form: $ax^2 + bx + c$

Example: Factor $6x^2 + 14x + 4$

- | | | |
|---|--|---|
| 1. Factor out the GCF | $6x^2 + 14x + 4$
$2(3x^2 + 7x + 2)$ | GCF is 2 |
| 2. Identify the values of a , b , and c for the expression $3x^2 + 7x + 2$ | $a = 3$; $b = 7$; and $c = 2$ | |
| 3. Find the product of $a \cdot c$ | $a \cdot c = 2 \cdot 3 = 6$ | |
| 4. List all possible factor pairs that equal $a \cdot c$ and identify the pair whose sum is b . | $\boxed{1 \cdot 6}$ $\boxed{1+6=7}$
$2 \cdot 3$ $2+3=5$ | 1 & 6 and 2 & 3 are the only factor pairs.
1 & 6 is the pair that adds up to 7 and multiplies up to 6. |
| 5. Substitute boxed values from Step 4 in for the bx term. | $3x^2 \boxed{+7x} + 2$
$3x^2 \boxed{+1x+6x} + 2$ | Substitute
$+1x + 6x$ for $+7x$ |
| 6. Group the two pairs | $\underline{3x^2 + 1x} + \underline{6x + 2}$ | First step in factor by grouping. |
| 7. Factor the GCF out of each pair. | $x(3x+1) + 2(3x+1)$ | The resulting common factor is $(3x+1)$ |
| 8. Factor out $(3x+1)$ | $(3x+1)(x+2)$ | These are the two factors of $3x^2 + 7x + 2$ |
| 9. Recall the original GCF from Step 1. | $2(3x^2 + 7x + 2) = 2(3x+1)(x+2)$ | There are 3 factors of $6x^2 + 14x + 4$ |
| Answer: | $2(3x+1)(x+2)$ | |
| 10. Check by multiplying. | $2(3x+1)(x+2) = 6x^2 + 14x + 4$ | |

FACTORING

Example: Factor $3y^2 - 4y - 4$

Note: If $a \cdot c$ is negative, the factor pairs have opposite signs.

- | | | | | | | | | | | | | | | | |
|------------------|--|--|---|--------------------|------------------|--------------------|-----------------|------------------|-----------------|------------------|-----------------|------------------|-----------------|------------------|---|
| 1. | Factor out the GCF. | | There is no GCF. | | | | | | | | | | | | |
| 2. | Identify the values of a , b , and c . | $a = 3$; $b = -4$; $c = -4$ | | | | | | | | | | | | | |
| 3. | Find the product of $a \cdot c$. | $ac = 3(-4) = -12$ | | | | | | | | | | | | | |
| 4. | List all possible factor pairs that equal $a \cdot c$ and identify the pair whose sum is b . | <table border="0" style="margin-left: 20px;"> <tr> <td>$+1 \cdot (-12)$</td> <td>$+1 + (-12) = -11$</td> </tr> <tr> <td>$-1 \cdot (+12)$</td> <td>$-1 + (+12) = +11$</td> </tr> <tr> <td style="border: 1px solid black;">$+2 \cdot (-6)$</td> <td style="border: 1px solid black;">$+2 + (-6) = -4$</td> </tr> <tr> <td>$-2 \cdot (+6)$</td> <td>$-2 + (+6) = +4$</td> </tr> <tr> <td>$+3 \cdot (-4)$</td> <td>$+3 + (-4) = -1$</td> </tr> <tr> <td>$-3 \cdot (+4)$</td> <td>$-3 + (+4) = +1$</td> </tr> </table> | $+1 \cdot (-12)$ | $+1 + (-12) = -11$ | $-1 \cdot (+12)$ | $-1 + (+12) = +11$ | $+2 \cdot (-6)$ | $+2 + (-6) = -4$ | $-2 \cdot (+6)$ | $-2 + (+6) = +4$ | $+3 \cdot (-4)$ | $+3 + (-4) = -1$ | $-3 \cdot (+4)$ | $-3 + (+4) = +1$ | Include all possible factor combinations, including the $+/-$ combinations since $a \cdot c$ is negative. |
| $+1 \cdot (-12)$ | $+1 + (-12) = -11$ | | | | | | | | | | | | | | |
| $-1 \cdot (+12)$ | $-1 + (+12) = +11$ | | | | | | | | | | | | | | |
| $+2 \cdot (-6)$ | $+2 + (-6) = -4$ | | | | | | | | | | | | | | |
| $-2 \cdot (+6)$ | $-2 + (+6) = +4$ | | | | | | | | | | | | | | |
| $+3 \cdot (-4)$ | $+3 + (-4) = -1$ | | | | | | | | | | | | | | |
| $-3 \cdot (+4)$ | $-3 + (+4) = +1$ | | | | | | | | | | | | | | |
| 5. | Substitute the boxed values from Step 4 in for the bx term. | $3y^2 - 4y - 4 = 3y^2 + 2y - 6y - 4$ | Substitute $2y - 6y$ in for $-4y$ | | | | | | | | | | | | |
| 6. | Group the two pairs. | $= \underline{3y^2 + 2y} - \underline{6y - 4}$ | First step in factor by grouping. | | | | | | | | | | | | |
| 7. | Factor the GCF out of each pair | $= y(3y + 2) - 2(3y + 2)$ | The resulting common factor is $(3y + 2)$. | | | | | | | | | | | | |
| 8. | Factor out $(3y + 2)$ | $(3y + 2)(y - 2)$ | These are the factors. | | | | | | | | | | | | |
| | Answer: | $(3y + 2)(y - 2)$ | | | | | | | | | | | | | |
| 9. | Check: | $(3y + 2)(y - 2) = 3y^2 - 4y - 4$ | | | | | | | | | | | | | |

FACTORIZING

Factoring Perfect Square Trinomials

Example: Factor $x^2 + 18x + 81$

STEP

1. Determine if the trinomial is a Perfect Square.

- Is the first term a perfect square? x^2
- Is the third term a perfect square? $+81$
- Is the second term twice the product of the square roots of the first and third terms? $+18x$

NOTES

Yes $x \cdot x = (x)^2$
Yes $9 \cdot 9 = (9)^2 = 81$
Yes $2(x \cdot 9) = 18x$

2. To factor a Perfect Square Trinomial:

- Find the square root of the first term x
- Identify the sign of the second term $+$
- Find the square root of the third term 9
- Write the factored form $x^2 + 18x + 81 = (x + 9)(x + 9) = (x + 9)^2$

Answer:

- Check your work $(x + 9)(x + 9) = x^2 + 18x + 81$

Example: Factor $25x^2 + 20xy + 4y^2$

STEP

1. Is the trinomial a Perfect Square Trinomial?

- Is the first term a perfect square? $25x^2$
- Is the third term a perfect square? $4y^2$
- Is the second term twice the product of the square roots of the first and third terms? $20xy$

NOTES

Yes $(5x)^2 = 25x^2$
Yes $(2y)^2 = 4y^2$
Yes $2(5x \cdot 2y) = 20xy$

2. Factor the trinomial

- Square root of the first term $\sqrt{25x^2} = 5x$
- Sign of the second term $+20xy \rightarrow +$
- Square root of the third term $\sqrt{4y^2} = 2y$
- Factored form $25x^2 + 20xy + 4y^2 = (5x + 2y)(5x + 2y) = (5x + 2y)^2$

Answer:

- Check your work $(5x + 2y)(5x + 2y) = 25x^2 + 20xy + 4y^2$

FACTORING

Factoring the Difference of Two Squares

Example: Factor $x^2 - 36$

STEP

1. Are there any common factors?
2. Determine if the binomial is the Difference of Two Squares

- Is the first term a perfect square? x^2
- Is the second term a perfect square? 36
- Is this the difference of the two terms? $x^2 \boxed{-} 36$

NOTES

No

Yes $x \cdot x = x^2$

Yes $6 \cdot 6 = 36$

Yes, “-” means the difference of the two terms.

3. To factor the Difference of Two Squares:

- Find the square root of the first term. x
- Find the square root of the second term. 6

$$\sqrt{x^2} = x$$

$$\sqrt{36} = 6$$

- Write the factored form. $x^2 - 36 = (x + 6)(x - 6)$

Answer:

$$(x + 6)(x - 6)$$

The factors are the product of the sum (+) and difference (-) of the terms' square roots.

- Check your work. $(x + 6)(x - 6) = x^2 - 36$

Example: Factor $36a^2 - 121b^2$

STEP

1. Are there any common factors?
2. Is the binomial a Difference of Two Squares?

- Is the first term a perfect square? $36a^2$
- Is the second term a perfect square? $121b^2$
- Is this the difference of the two terms? $36a^2 \boxed{-} 121b^2$

NOTES

No

Yes $6a \cdot 6a = 36a^2$

Yes $11b \cdot 11b = 121b^2$

Yes, “-” means the difference of the two terms.

3. To factor the Difference of Two Squares:

- Find the square root of the first term. $6a$
- Find the square root of the second term. $11b$

$$\sqrt{36a^2} = 6a$$

$$\sqrt{121b^2} = 11b$$

- Write the factored form. $36a^2 - 121b^2 = (6a + 11b)(6a - 11b)$

Answer:

$$(6a + 11b)(6a - 11b)$$

The factors are the product of the sum (+) and difference (-) of the terms' square roots.

- Check your work. $(6a + 11b)(6a - 11b) = 36a^2 - 121b^2$

FACTORING

FACTORING STRATEGY

1. Is there a common factor? If so, factor out the GCF, or the opposite of the GCF so that the leading coefficient is positive.

2. How many terms does the polynomial have?

If it has *two terms*, look for the following problem type:

- a. The difference of two squares

If it has *three terms*, look for the following problem types:

- a. A perfect-square trinomial
- b. If the trinomial is not a perfect square, use the grouping method.

If it has *four or more terms*, try to factor by grouping.

3. Can any factors be factored further? If so, factor them completely.

4. Does the factorization check? Check by multiplying.

FACTORING STRATEGY

Always check for

Greatest Common Factor First

Then

continue:

2 – Terms

Difference of Two Squares

$$\begin{aligned}x^2 - 4 \\(x)^2 - (2)^2 \\(x+2)(x-2)\end{aligned}$$

Perfect Square Trinomials

$$\begin{aligned}4x^2 + 20x + 25 \\(2x)^2 + 2(2x)(5) + 5^2 \\(2x+5)(2x+5) \\(2x+5)^2\end{aligned}$$

3 – Terms

Form: $x^2 + bx + c$

$$x^2 - 6x + 5$$

List factors of 5

$$5 \cdot 1$$

$$\textcircled{-5 \cdot -1}$$

Choose the pair which adds to -6

$$-5 + -1 = -6$$

$$(x-5)(x-1)$$

Form: $ax^2 + bx + c$

$$5x^2 - 7x - 6$$

$$5 \cdot -6 = -30$$

List factors of -30

$$-1 \cdot 30 \quad 1 \cdot -30$$

$$-2 \cdot 15 \quad 2 \cdot -15$$

$$-3 \cdot 10 \quad \textcircled{3 \cdot -10}$$

$$-5 \cdot 6 \quad 5 \cdot -6$$

Choose the pair which adds to -7

$$3x + -10x = -7x$$

Substitute this pair in for the middle term.

$$5x^2 + 3x - 10x - 6$$

Factor by Grouping.

$$x(5x+3) - 2(5x+3)$$

$$(5x+3)(x-2)$$

4 – Terms

Factor by Grouping

$$\begin{aligned}7x^2 + 14x - 6x - 12 \\7x(x+2) - 6(x+2) \\(x+2)(7x-6)\end{aligned}$$

FACTORING

Problems

Factor completely.

1. $x^2 + 6x + 8$

2. $u^2 + 15u + 56$

3. $x^2 + 8x - 20$

4. $y^2 - 6y - 40$

5. $m^2 - 15m + 54$

6. $x^2 + 5xy - 14y^2$

7. $a^2 + 16ab + 28b^2$

8. $x^3 - 3x^2 - 18x$

9. $14x^2 - x - 3$

10. $3x^2 - 19x + 20$

Answers

$(x + 2)(x + 4)$

$(u + 8)(u + 7)$

$(x + 10)(x - 2)$

$(y - 10)(y + 4)$

$(m - 9)(m - 6)$

$(x + 7y)(x - 2y)$

$(a + 14b)(a + 2b)$

$x(x - 6)(x + 3)$

$(7x + 3)(2x - 1)$

$(x - 5)(3x - 4)$

FACTORING

Problems (continued)

Answers

11. $2x^2 - 16x - 96$

$$2(x-12)(x+4)$$

12. $54x^5 - 54x^4 - 108x^3$

$$54x^3(x-2)(x+1)$$

13. $6x^2 + 2x + 12$

$$2(3x^2 + x + 6)$$

14. $36x^2 + 18x - 54$

$$18(2x+3)(x-1)$$

15. $x^2 - 1$

$$(x-1)(x+1)$$

16. $16x^4 - 81$

$$(4x^2 + 9)(2x+3)(2x-3)$$

17. $27x^4 - 48x^2$

$$3x^2(3x+4)(3x-4)$$

18. $6x^2 + 33x + 45$

$$3(2x+5)(x+3)$$

19. $x^2 - 18x + 81$

$$(x-9)^2$$

20. $9x^2 + 24x + 16$

$$(3x+4)^2$$

21. $20x^2 + 60x + 45$

$$5(2x+3)^2$$

SOLVING LINEAR EQUATIONS

To Solve a Linear Equation

1. Remove grouping symbols by using the distributive property.
2. Combine like terms to simplify each side.
3. Clear fractions by multiplying both sides of the equation by the Least Common Denominator.
4. Move the variable terms to one side and the constants to the other side. Do this by adding or subtracting terms.
5. Solve for the variable by multiplying by the inverse or dividing by the coefficient of x .
6. Check by substituting the result into the original equation.

Example Solve $3(x-4)-(2x+8)=4x+4$

$$3x-12-2x-8=4x+4$$

$$x-20=4x+4$$

$$-3x=24$$

$$\frac{-3x}{-3} = \frac{24}{-3}$$

Solution: $x = -8$

Use the distributive property.

Combine terms

Move variable to one side, numbers to the other

Divide to solve for the variable.

Check: $3(-8-4)-(2(-8)+8)=4(-8)+4$

$$3(-12)-(-16+8)=-32+4$$

$$-36-(-8)=-28$$

$$-28=-28$$

Example: Solve $\frac{2}{3}x - \frac{1}{2} = \frac{3}{4}$

$$12\left(\frac{2}{3}x\right) - 12\left(\frac{1}{2}\right) = 12\left(\frac{3}{4}\right)$$

$$8x - 6 = 9$$

$$\frac{8x}{8} = \frac{15}{8}$$

Solution: $x = \frac{15}{8}$

Clear fractions by multiplying by the LCD

Check: $\frac{2}{3}\left(\frac{15}{8}\right) - \frac{1}{2} = \frac{3}{4}$

$$\cancel{2}^1\left(\frac{\cancel{1}5^5}{\cancel{8}_4}\right) - \frac{1}{2} = \frac{3}{4}$$

$$\frac{5}{4} - \frac{2}{4} = \frac{3}{4}$$

$$\frac{3}{4} = \frac{3}{4}$$

SOLVING LINEAR EQUATIONS

Special Cases:

1. When the variable terms drop out and the result is a true statement, (i.e., $2 = 2$ or $0 = 0$), there are an **infinite number of solutions**. The equation is called an **identity**.

Example: Solve $2 + 9x = 3(3x + 1) - 1$
 $2 + 9x = 3(3x + 1) - 1$
 $2 + 9x = 9x + 3 - 1$
 $2 + 9x = 9x + 2$
 $0 = 0$ Infinite number of solutions

Solution: **All real numbers**

2. When the variable terms drop out and the result is a false statement (i.e., $10 = 2$ or $8 = 0$), there is **no solution**. The equation is called a **contradiction**.

Example: Solve $3x - 5 = 3(x - 2) + 4$
 $3x - 5 = 3x - 6 + 4$
 $3x - 5 = 3x - 2$
 $-5 = -2$

Solution: **No solution**

SOLVING LINEAR EQUATIONS

Problems

Solve.

1. $3 - (2x + 5) = x - 3(2x - 5)$

$$x = \frac{17}{3}$$

2. $(\frac{1}{2})(x - 5) = (\frac{1}{3})(x + 2)$

$$x = 19$$

3. $2x + 3 = 3x + 6 - x + 4$

No solution

4. $2x + 3 = 3x + 6 - x - 3$

Infinite number of solutions:
All real numbers

5. $3x + 4 - 5x + 2 = 2(3 + x)$

$$x = 0$$

SOLVING LINEAR INEQUALITIES

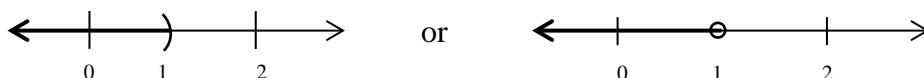
Solving Linear Inequalities

Linear inequalities are solved almost exactly like linear equations. There is only one exception: if it is necessary to divide or multiply by a **NEGATIVE** number, the inequality sign must be reversed.

The solution can be written as a statement of inequality or in interval notation. It can also be shown as a graph. In interval notation and graphing:

- Use a bracket, [], or closed circle, if the endpoint **is** included in the solution
- Use parenthesis, (), or open circle, if the endpoint **is not** included in the solution

Example: $x < 1$ **Interval Notation:** $(-\infty, 1)$



Example: $-3(x-2) \leq x-5$
 $-3x+6 \leq x-5$
 $-4x \leq -11$

Distribute

Move variable term to one side,
 constants to the other

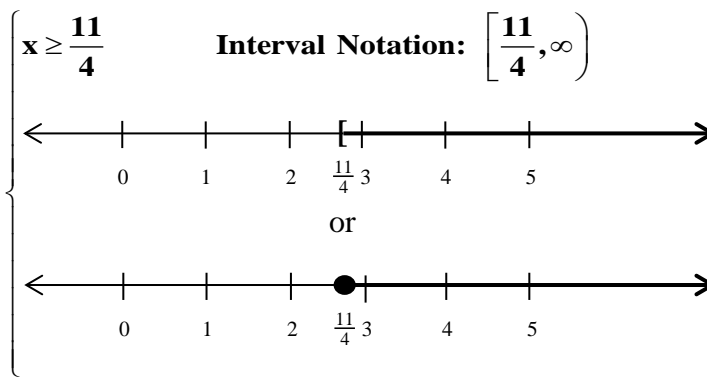
$$\frac{-4x}{-4} \geq \frac{-11}{-4}$$

Divide by -4 and reverse the
 inequality because of division -4

$$x \geq \frac{11}{4}$$

Simplify

Solution:



SOLVING LINEAR INEQUALITIES

Problems

Solve.

1. $4a - 7 > 3(2a + 5) - 2$

$a < -10$ or $(-\infty, -10)$

2. $2(5x + 3) - 3 \leq 6(2x - 3) + 15$

$x \geq 3$ or $[3, \infty)$

3. $3x + 4 - 5x + 2 < 2(3 + x)$

$x > 0$ or $(0, \infty)$

4. $10 - 4x + 8 \geq 6 - 2x + 10$

$x \leq 1$ or $(-\infty, 1]$

5. $5(x + 3) - 6x \leq 3(2x + 1) - 4x$

$x \geq 4$ or $[4, \infty)$

6. $2(x - 5) + 3x < 4(x - 6) + 3$

$x < -11$ or $(-\infty, -11)$

SOLVING LINEAR INEQUALITIES

Compound Inequalities

Compound inequalities are solved the same way as simple inequalities. Any operation (addition, subtraction, multiplication, division) must be performed on all three pieces of the inequality.

Never remove the variable from the middle piece of the inequality. Always remember that when dividing or multiplying by a **NEGATIVE** number, the inequality sign must be reversed.

Example: Solve $-8 < 3x - 2 \leq 4$

$$\begin{array}{c} +2 \quad +2 \quad +2 \\ -6 < 3x \leq 6 \end{array}$$

Add 2 to all three parts of the inequality

$$\begin{array}{c} -6 \quad 3x \quad 6 \\ \frac{-6}{3} < \frac{3x}{3} \leq \frac{6}{3} \end{array}$$

Divide all three parts of the inequality by 3

$$-2 < x \leq 2$$

Simplify

In interval notation, this is $(-2, 2]$, because x is between -2 and 2 , including the endpoint 2 , but not including the endpoint -2 .

Solution: $\left\{ \begin{array}{l} -2 < x \leq 2 \quad \text{Interval Notation: } (-2, 2] \\ \leftarrow \begin{array}{c} | \quad | \quad | \quad | \quad | \\ -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \end{array} \rightarrow \quad \text{or} \quad \leftarrow \begin{array}{c} | \quad | \quad | \quad | \quad | \\ -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \end{array} \rightarrow \end{array} \right.$

Example: Solve $-5 < -3x < 12$

$$\begin{array}{c} -5 \quad -3x \quad 12 \\ -\frac{5}{-3} > \frac{-3x}{-3} > \frac{12}{-3} \end{array}$$

When dividing an inequality by a negative number, reverse the inequality sign.

$$\begin{array}{c} 5 \quad x \quad -4 \\ \frac{5}{3} > x > -4 \end{array}$$

Keep the variable in the middle piece of the inequality.

$$-4 < x < \frac{5}{3}$$

Arrange the inequality so that the lesser value is on the left and the greater value on the right, and change the inequality symbols to preserve the relationship.

$$\left(-4, \frac{5}{3}\right)$$

Solution: $\left\{ \begin{array}{l} -4 < x < \frac{5}{3} \quad \text{Interval Notation: } \left(-4, \frac{5}{3}\right) \\ \leftarrow \begin{array}{c} | \quad | \quad | \quad | \quad | \\ -5 \quad -4 \quad 0 \quad \frac{5}{3} \quad 2 \end{array} \rightarrow \quad \text{or} \quad \leftarrow \begin{array}{c} | \quad | \quad | \quad | \quad | \\ -5 \quad -4 \quad 0 \quad \frac{5}{3} \quad 2 \end{array} \rightarrow \end{array} \right.$

SOLVING LINEAR INEQUALITIES

Problems

Answers

Solve

- | | | | | |
|-----|--------------------------|-------------------------------------|----|-------------------------------|
| 1. | $-2 < 3x + 1 < 4$ | $-1 < x < 1$ | or | $(-1, 1)$ |
| 2. | $-5 \leq 4x + 5 \leq 5$ | $-\frac{5}{2} \leq x \leq 0$ | or | $[-\frac{5}{2}, 0]$ |
| 3. | $-4 < 4x + 2 \leq 8$ | $-\frac{3}{2} < x \leq \frac{3}{2}$ | or | $(-\frac{3}{2}, \frac{3}{2}]$ |
| 4. | $3 < 5 - 2x < 7$ | $-1 < x < 1$ | or | $(1, 1)$ |
| 5. | $2 \leq 1 - x < 6$ | $-5 < x \leq -1$ | or | $(-5, -1]$ |
| 6. | $-2 < 4 - 3x < 0$ | $\frac{4}{3} < x < 2$ | or | $(\frac{4}{3}, 2)$ |
| 7. | $0 < 1 - 4x < 7$ | $-\frac{3}{2} < x < \frac{1}{4}$ | or | $(-\frac{3}{2}, \frac{1}{4})$ |
| 8. | $-4 \leq 3 - 2x \leq -1$ | $2 \leq x \leq \frac{7}{2}$ | or | $[2, \frac{7}{2}]$ |
| 9. | $-7 < 4 - 2x < -4$ | $4 < x < \frac{11}{2}$ | or | $(4, \frac{11}{2})$ |
| 10. | $4 < 4 - 3x < 6$ | $-\frac{2}{3} < x < 0$ | or | $(-\frac{2}{3}, 0)$ |

GRAPHING LINEAR EQUATIONS

Standard Form: $ax + by = c$

A line is made up of an infinite number of points in the form (x,y) . The coordinates for each of these points will satisfy the equation for the line (make it true).

Two special points on the line are its **x and y intercepts**. These are the points on the line where the line crosses the x axis and the y axis. The x intercept is in the form $(x,0)$, and the y intercept is in the form $(0,y)$.

Find the **x intercept** of the line by substituting 0 for y, and solving for x, $(x,0)$.

Find the **y intercept** of the line by substituting 0 for x, and solving for y, $(0,y)$.

Example: Graph the line using the intercept method $3x - 4y = -12$

Find the x intercept:

Substitute 0 for y and solve for x:

$$\begin{aligned}3x - 4y &= -12 \\3x - 4(0) &= -12 \\3x - 0 &= -12 \\3x &= -12 \\3x &= \frac{-12}{3} \\x &= -4\end{aligned}$$

The x intercept is **$(-4,0)$** .

Find the y intercept:

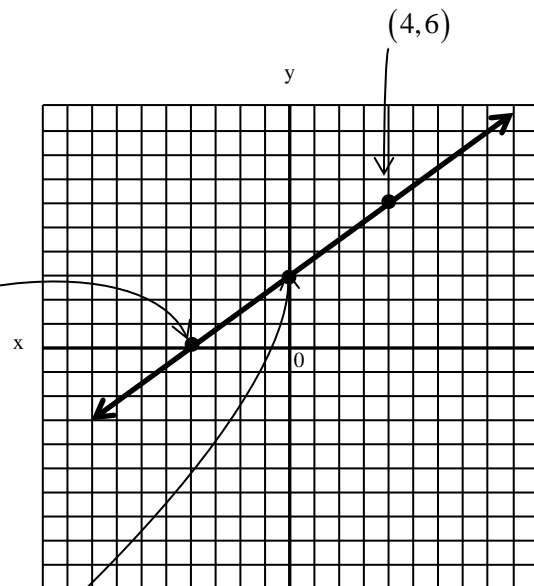
Substitute 0 for x and solve for y:

$$\begin{aligned}3x - 4y &= -12 \\3(0) - 4y &= -12 \\0 - 4y &= -12 \\-4y &= -12 \\ \frac{-4y}{-4} &= \frac{-12}{-4} \\y &= 3\end{aligned}$$

The y intercept is **$(0,3)$** .

To graph the line, graph these two points, and connect them.

As a check, find one more point on the line. Let $x = 4$ and find $y = 6$. The point $(4,6)$ is on the line and collinear with $(-4,0)$ and $(0,3)$.



GRAPHING LINEAR EQUATIONS

Slope—Intercept Form: $y = mx + b$

If the equation of the line is in slope-intercept form, we use the slope m , and the y intercept, $(0,b)$, to graph the line.

The **slope** of a line passing through points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1} \text{ if } x_2 \neq x_1$$

The slope calculation is called a Rate of Change. The rate of change describes how one quantity (the numerator) changes with respect to another (the denominator).

Example: Graph the line: $y = 2x + 3$.

Identify the slope as a fraction:

In the equation $y = mx + b$, m is the slope.

In the equation $y = 2x + 3$, 2 is the slope, so $m = \frac{2}{1}$.

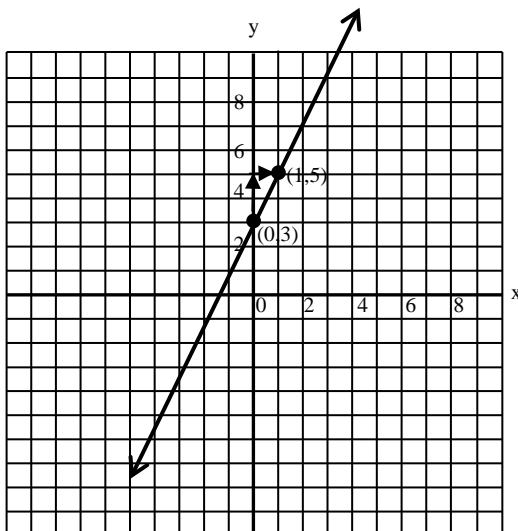
Identify the y intercept as a point:

In the equation, $y = mx + b$, $(0,b)$ is the y intercept.

In the equation, $y = 2x + 3$, $(0,3)$ is the y intercept

Graph the y intercept:

$(0,3)$ is the first point to be graphed



Graph the slope:

$$m = \frac{2}{1} = \frac{\text{rise of 2}}{\text{run of 1}}$$

2 is positive; move up 2 (rise) from the y intercept, $(0,3)$, to the point $(0,5)$.

1 is positive; then move 1 to the right (run) from $(0,5)$ to $(1,5)$.

$(1,5)$ is the second point of the line.

Graph the line through the two points:

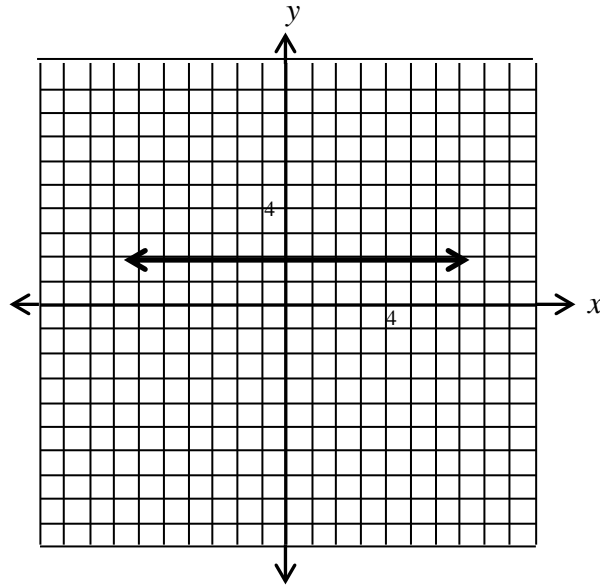
Graph the line through the points $(0,3)$ and $(1,5)$.

GRAPHING LINEAR EQUATIONS

Special Cases:

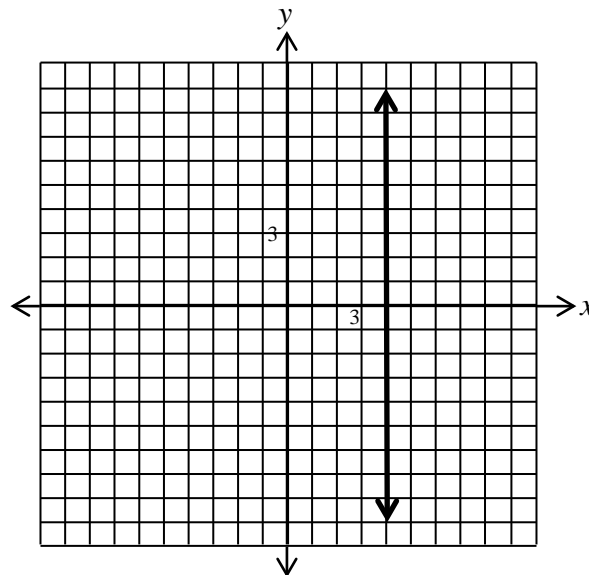
The graph of the linear equation $y = k$, where k is a real number, is the horizontal line going through the $(0, k)$

Example: $y = 2$



The graph of the linear equation $x = k$, where k is a real number, is the vertical line going through the point $(k, 0)$.

Example: $x = 4$

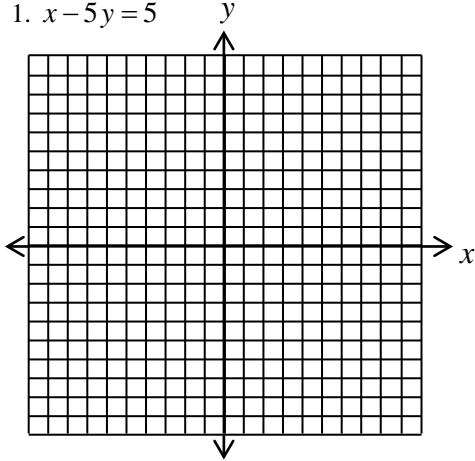


GRAPHING LINEAR EQUATIONS

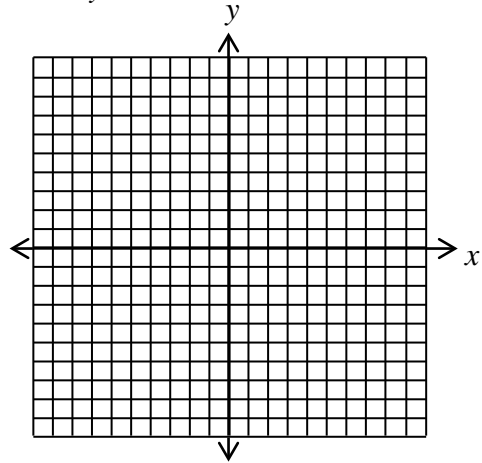
Problems

Graph the following equations:

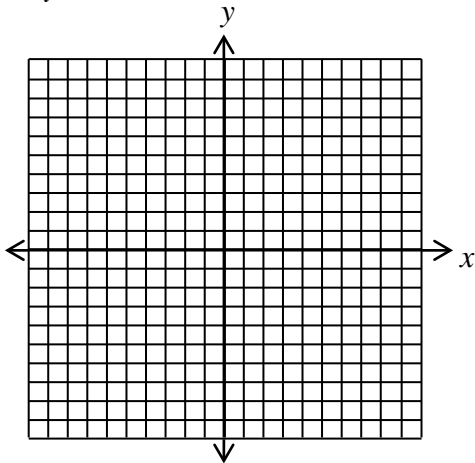
1. $x - 5y = 5$



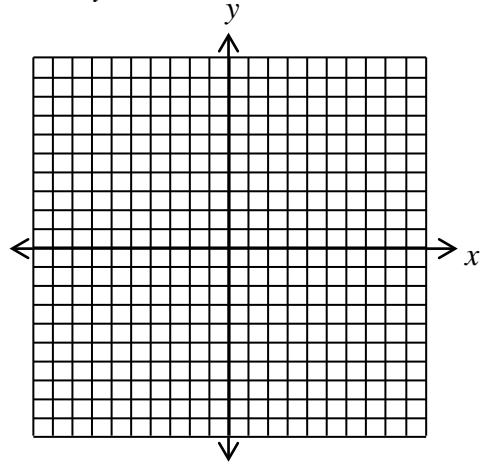
2. $x + 3y = 5$



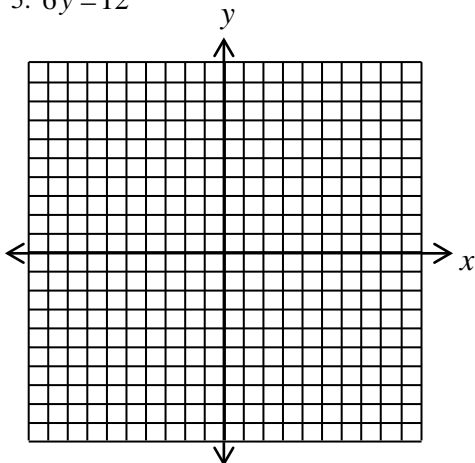
3. $y = x + 4$



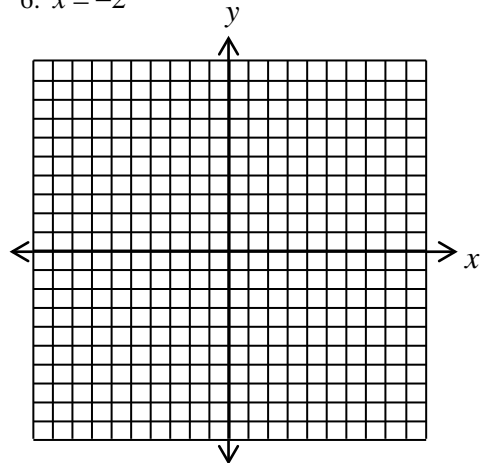
4. $x = 2y$



5. $6y = 12$



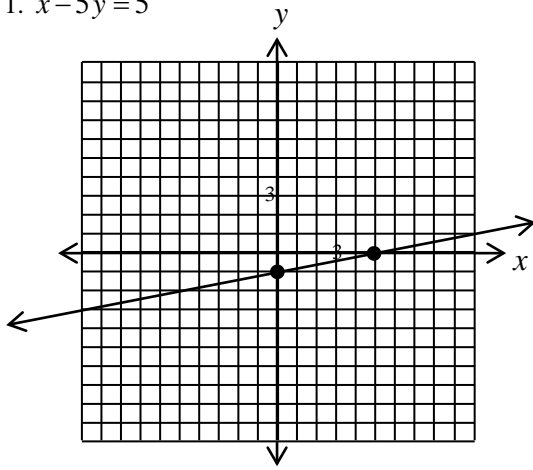
6. $x = -2$



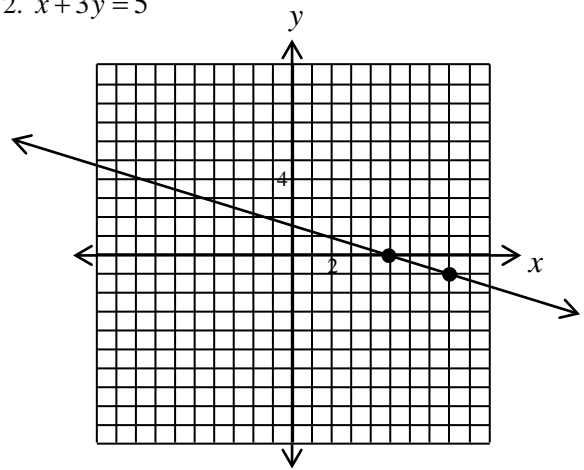
GRAPHING LINEAR EQUATIONS

Answers

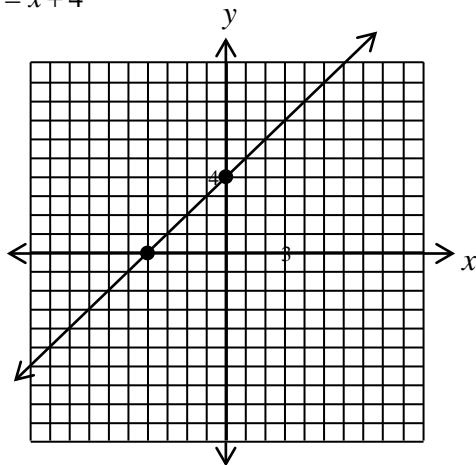
1. $x - 5y = 5$



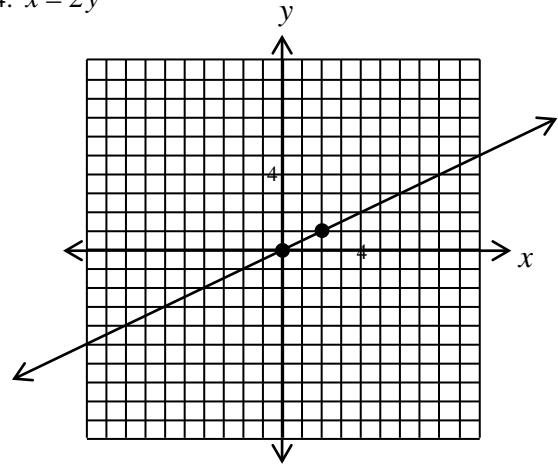
2. $x + 3y = 5$



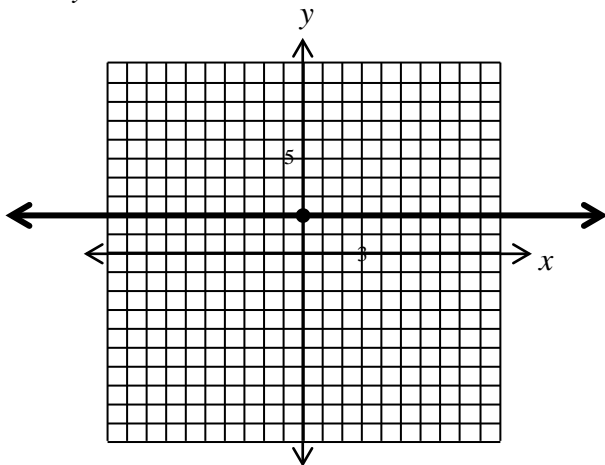
3. $y = x + 4$



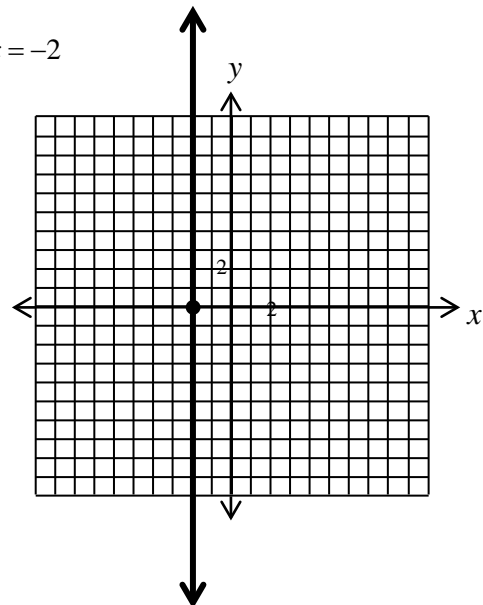
4. $x = 2y$



5. $6y = 12$



6. $x = -2$



FINDING THE EQUATION OF A LINE

The standard form of the equation of a line is written as

$$ax + by = c$$

where a and b are not both 0.

Definition of slope: Let L be a line passing through the points (x_1, y_1) and (x_2, y_2) . Then the slope of L is given by the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope-intercept form of the equation of a line is written as

$$y = mx + b$$

where m is the slope and the point $(0, b)$ is the y-intercept.

Point-Slope Form: The equation of the straight line passing through (x_1, y_1) and having slope m is given by $y - y_1 = m(x - x_1)$.

Parallel Property: Parallel lines have the same slope.

Perpendicular Property: When two lines are perpendicular, their slopes are negative (opposite) reciprocals of one another. The product of their slopes is -1 .

FINDING THE EQUATION OF A LINE

To find the equation of a line, use: **Slope-Intercept form:** $y = mx + b$, slope m , y intercept $(0, b)$
or **Point-Slope form:** $y - y_1 = m(x - x_1)$, slope m , point (x_1, y_1)

Slope-Intercept Form

When given the **slope** and **y intercept:** use the **slope-intercept form** $y = mx + b$

Example: Find the equation of the line with slope $\frac{3}{4}$ and y intercept $(0, -2)$

Use the slope-intercept form, $y = mx + b$, and fill in the values:

Solution: The equation of the line is:

$$y = \frac{3}{4}x - 2$$

Make the substitutions of

$$m = \frac{3}{4} \left(\text{slope is } \frac{3}{4} \right) \text{ and}$$

$$b = -2 \left(y \text{ intercept is } (0, -2) \right)$$

Point-Slope Form

When given the **slope** and a **point** on the line: use the **point-slope form** $y - y_1 = m(x - x_1)$

Example: Find the equation of the line with slope $\frac{2}{3}$, through the point $(6, 1)$

Use the point-slope form, $y - y_1 = m(x - x_1)$, and fill in the values.

$$y - 1 = \frac{2}{3}(x - 6)$$

$$y - 1 = \frac{2}{3}x - \frac{2}{3} \cdot \frac{6}{1}$$

$$y - 1 = \frac{2}{3}x - 4$$

$$y - 1 + 1 = \frac{2}{3}x - 4 + 1$$

$$y = \frac{2}{3}x - 3$$

Make the substitutions of $m = \frac{2}{3}$ (slope is $\frac{2}{3}$) and

$x_1 = 6$ and $y_1 = 1$ (point $(6, 1)$).

Solution: The equation of the line is

$$y = \frac{2}{3}x - 3$$

FINDING THE EQUATION OF A LINE

When given two points on the line, first find the slope m , then use the **slope** and either one of the **points** to find the equation using the **point-slope form** $y - y_1 = m(x - x_1)$

Example: Find the equation of the line through the points (2,3) and (1,1).

$$\text{Slope } m = \frac{3-1}{2-1} = \frac{2}{1} = 2$$

$$\begin{array}{l} y - y_1 = m(x - x_1) \\ y - 3 = 2(x - 2) \leftarrow \left\{ \begin{array}{l} \text{Make the substitutions of} \\ m = 2(\text{slope is } 2) \text{ and } x_1 = 2 \text{ and } y_1 = 3 \\ (\text{point}(2,3)). \end{array} \right. \\ y - 3 = 2x - 4 \\ y = 2x - 1 \end{array}$$

Solution: The equation of the line is $y = 2x - 1$

FINDING THE EQUATION OF A LINE

Problems

Answers

1. Put the following in slope-intercept form.

a. $8x - 4y = 4$

$$y = 2x - 1$$

b. $2x + 3y = 3$

$$y = -\frac{2}{3}x + 1$$

2. Find the slope.

a. $(3,4)$ and $(7,9)$

$$\frac{5}{4}$$

b. $(-2,1)$ and $(3,-3)$

$$-\frac{4}{5}$$

3. Find the equation of the line through $(5,3)$
and parallel to $y = -2x + 1$.

$$y = -2x + 13$$

4. Find the equation of the line through $(-1,-3)$
and perpendicular to $y = 4x - 2$.

$$y = -\frac{1}{4}x - \frac{13}{4}$$

FINDING THE EQUATION OF A LINE

Problems

Find the equation of the line that satisfies the given conditions.
Write the equation in both standard form and slope-intercept form.

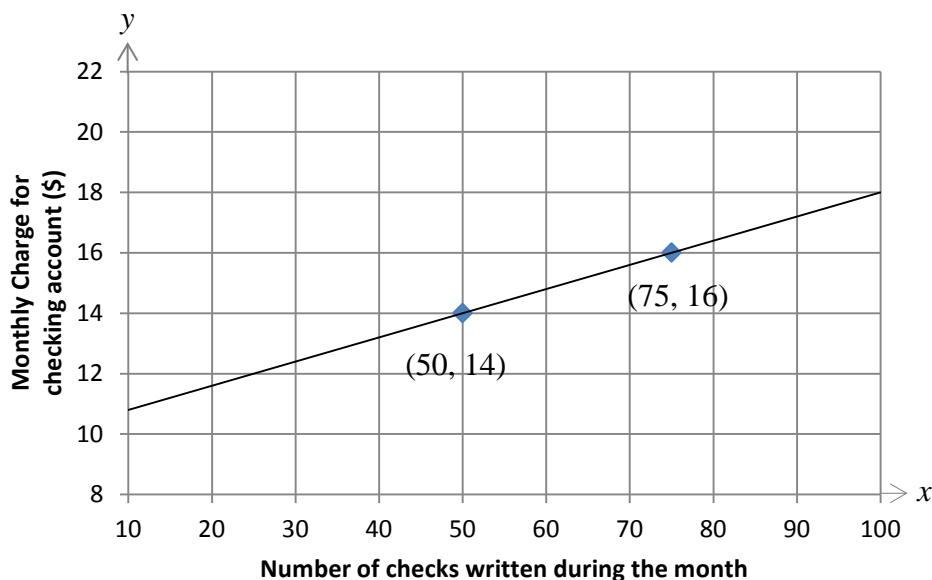
Answers

		<u>Standard Form</u>	<u>Slope-Intercept Form</u>
5. slope = $\frac{1}{2}$	line passes through (3,4)	$x - 2y = -5$	$y = \frac{1}{2}x + \frac{5}{2}$
6. slope = $-\frac{5}{6}$	line passes through (0,0)	$5x + 6y = 0$	$y = -\frac{5}{6}x$
7. slope = 1	y-intercept -3	$x - y = 3$	$y = x - 3$
8. horizontal line through (1,4)		$y = 4$	
9. slope is undefined and passing through (-5,6)		$x = -5$	
10. slope = $-\frac{3}{2}$	x-intercept -5	$3x + 2y = -15$	$y = -\frac{3}{2}x - \frac{15}{2}$
11. horizontal line through (5,-3)		$y = -3$	
12. vertical line passing through (5,-4)		$x = 5$	
13. line passing through (1,2) and (5,4)		$x - 2y = -3$	$y = \frac{1}{2}x + \frac{3}{2}$
14. line passing through (-3,4) and (5,-1)		$5x + 8y = 17$	$y = -\frac{5}{8}x + \frac{17}{8}$
15. line passing through (4,-3) and (4,-7)		$x = 4$	
16. line parallel to $3x + y = 6$ and passing through (1,2)		$3x + y = 5$	$y = -3x + 5$
17. line passing through (5,-3) and parallel to $y - 2 = 0$		$y = -3$	
18. line perpendicular to $2x + 5y = 3$ and passing through (1,7)		$5x - 2y = -9$	$y = \frac{5}{2}x + \frac{9}{2}$

APPLICATION OF LINEAR EQUATIONS

Example: Determine Rate of Change

A credit union offers a checking account with a service charge for each check written. The relationship between the monthly charge for each check “y” and the number of checks written “x” is graphed below. At what rate does the monthly charge for the checking account change? Also, find the unit cost, another way to express the rate of change.



Find the rate of change (slope of the line). The units will be dollars per number of checks.

From the graph, we see that two points on the line are $(50, 14)$ and $(75, 16)$. If we let $(x_1, y_1) = (50, 14)$ and $(x_2, y_2) = (75, 16)$, we have

$$\text{Rate of change} = \frac{(y_2 - y_1)\text{dollars}}{(x_2 - x_1)\text{checks}} = \frac{(16 - 14)\text{dollars}}{(75 - 50)\text{checks}} = \frac{2\text{dollars}}{25\text{checks}}$$

The rate of change can be expressed as \$2 for every 25 checks.

Rate of change can also be expressed as \$0.08 per check or 8¢ per check.

The monthly cost of the checking account increases \$2 for every 25 checks written. To find the cost of 1 check (unit cost), take the fraction which represents the rate of change, $\frac{2}{25}$, and divide both the numerator and denominator by 25.

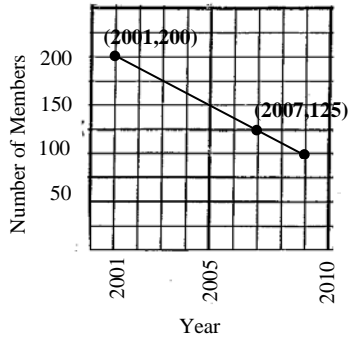
$$\frac{2\text{ dollars}}{25\text{ checks}} = \frac{2 \div 25}{25 \div 25} = \frac{.08\text{ dollars}}{1\text{ check}} = \$.08 / \text{check.}$$

The unit cost = \$.08 per check.

APPLICATION OF LINEAR EQUATIONS

Problems

The graph models the number of members in an organization from 2001 to 2010.



Answers

How many members did the organization have in 2009?

100 members

Find the rate of change.
(slope of the line)

loss of 25 members every
two years

What is the rate of change per year?

loss of 12.5 members per
year

FUNCTIONS

Recall that **relation** is a set of ordered pairs and that a **function** is a special type of relation.

A **function** is a set of ordered pairs (a relation) in which to each first component, there corresponds exactly one second component.

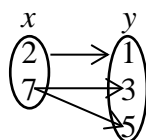
The set of first components is called the **domain of the function** and the set of second components is called **the range of the function**.

Examples: Determine if the following are functions.

Domain	Range
x	y
-4	0
-8	1
-16	2

This is a function, since for each first component, there is exactly one second component.

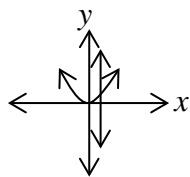
Domain Range



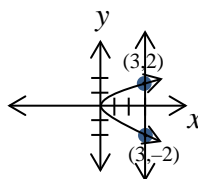
This is not a function since for the first component, 7, there are two different second components, 3 and 5.

Vertical Line Tests: A graph in the plane represents a function if no vertical line intersects the graph at more than one point.

Examples:



This is a graph of a function. It passes the vertical line test.



This is not a graph of a function. For example, the x -value of 3 is assigned two different y values, 2 and -2 . (for the first component, 3, there are two different second components, 2 and -2 .)

Since we will often work with sets of ordered pairs of the form (x, y) , it is helpful to define a function using the variables x and y .

Given a relation in x and y , if to each value of x in the domain there corresponds exactly one value of y in the range, then y is said to be a **function of x** .

FUNCTIONS

Notation: To denote that y is a function of x , we write $y = f(x)$. The expression “ $f(x)$ ” is read f of x . It does not mean f times x . Since y and $f(x)$ are equal, they can be used interchangeably. This means we can write $y = x^2$, or we can write $f(x) = x^2$.

Evaluate: To evaluate or calculate a function, replace the x in the function rule by the given x value from the domain and then compute according to the rule. For example:

Examples:

1. Given: $f(x) = 6x + 5$

Find: $f(2) = 6(2) + 5 = 17$

$$f(0) = 6(0) + 5 = 5$$

$$f(-1) = 6(-1) + 5 = -1$$

2. Given: $g(x) = 3x^2 - 5x + 8$

Find: $g(1) = 3(1)^2 - 5(1) + 8 = 6$

$$g(0) = 3(0)^2 - 5(0) + 8 = 8$$

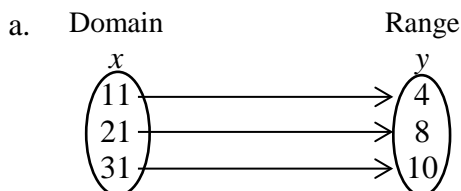
$$g(-1) = 3(-1)^2 - 5(-1) + 8 = 16$$

FUNCTIONS

Problems

Answers

1. Determine whether or not each relation defines a function. If no, explain why not.



a. Yes, for each first component, there corresponds exactly one second component.

b. Domain Range

x	y
3	2
5	6
3	7

b. No, for each first component, 3, there corresponds two different second components, 2 and 7.

c. $\{(1,3),(2,-4),(1,0)\}$

c. No, for the first component, 1, there corresponds two different second components 3 and 0.

d. $\{(4,3),(-2,3),(1,3)\}$

d. Yes, for each first component there corresponds exactly one second component.

2. Determine the domain and range of each of the following:

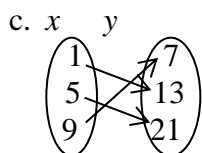
a. $\{(-3,4),(4,2),(0,0),(-2,7)\}$

a. Domain: $\{-3,-2,0,4\}$
Range: $\{0,2,4,7\}$

b.

x	y
-5	2
-7	4
-9	6

b. Domain: $\{-5,-7,-9\}$
Range: $\{2,4,6\}$



c. Domain: $\{1,5,9\}$
Range: $\{7,13,21\}$

3. Given $f(x) = x + 2$ and $g(x) = x - 3$, find the following:

a. $f(-2)$

a. 0

b. $f(-4)$

b. -2

c. $g(0)$

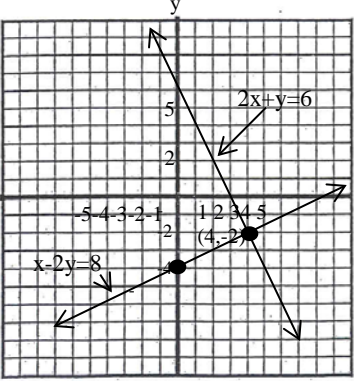
c. -3

d. $g(2)$

d. -1

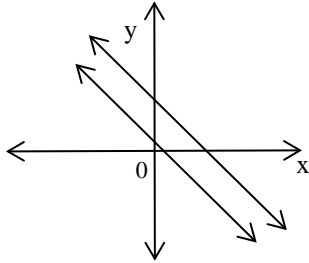
SYSTEMS OF LINEAR EQUATIONS

The following summary compares the graphing, elimination (addition), and substitution methods for solving linear systems of equations.

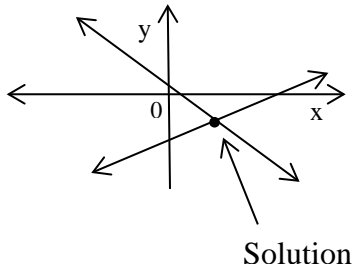
Method	Example	Notes
Graphing	$2x + y = 6$ $x - 2y = 8$ 	<p>You can see the solution is where the two lines intersect but if the solution does not involve integers it's impossible to tell exactly what the solution is.</p> <p>The solution for this system of linear equations is $(4, -2)$.</p>
Substitution	$\begin{cases} y = 3x - 1 \\ 3x - 2y = -4 \end{cases}$ <p>Substitute $3x - 1$ for y:</p> $3x - 2(3x - 1) = -4$ <p>Solve for x: $x = 2$</p> <p>Back-substitute: $y = 3(2) - 1 = 5$</p> <p>Solution: $(2, 5)$</p>	<p>Gives exact solutions.</p> <p>The solution for this system of equations is $(2, 5)$.</p>
Elimination (Addition)	$\begin{cases} 2x + 3y = -8 \\ 5x + 4y = -34 \end{cases}$ <p>Multiply the top equation by 5 and the bottom equation by -2. This will result in opposite coefficients of x.</p> $\begin{cases} 10x + 15y = -40 \\ -10x - 8y = 68 \end{cases}$ <p>Add the two equations: $7y = 28$</p> $y = 4$ <p>To find x, substitute $y = 4$ into either original equation.</p> $2x + 3(4) = -8$ $x = -10$ <p>Solution: $(-10, 4)$</p>	<p>Gives exact solution.</p> <p>Easy to use if a variable is on one side by itself. The solution for this system of equations is $(-10, 4)$.</p>

SYSTEMS OF LINEAR EQUATIONS

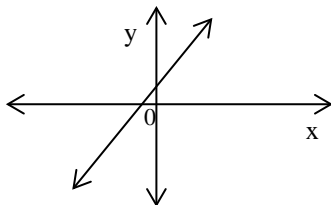
Three Possible Solution Types



No Solution (Parallel Lines)
Inconsistent System
Independent Equations
= different # (contradiction)



Exactly One Solution (point of intersection)
Consistent System
Independent Equations



Infinitely Many Solutions
(Lines Coincide-any point on the line is a solution)
Consistent System
Dependent Equations
= same # (identity)

SYSTEMS OF LINEAR EQUATIONS

Solving Linear Systems By Substitution Method

To solve a linear system of two equations in two variables by substitution:

1. Solve one of the equations for one of the variables.
2. Substitute the expression obtained in step 1 for that variable in the other equation, and solve the resulting equation in one variable.
3. Substitute the value for that variable into one of the original equations and solve for the other variable.
 - a. if we get $0 = \text{nonzero number}$ or $\# = \text{different } \#$ (contradiction), the system is inconsistent, the lines are parallel, the equations are independent, and there is no solution.
 - b. if we get $0 = 0$ or $\# = \text{same } \#$ (identity), the system is consistent, the lines are the same (coincide), the equations are dependent, and there are infinitely many solutions.

Example:

Solve the system of linear equations using the substitution method.

$$\begin{cases} 3x + 5y = 3 \\ x = 8 - 4y \end{cases}$$

This equation is solved for x .

$$3(8 - 4y) + 5y = 3$$

Substitute $8 - 4y$ in for x in the first equation.

$$\left. \begin{array}{l} 24 - 12y + 5y = 3 \\ 24 - 7y = 3 \\ -7y = -21 \\ y = 3 \end{array} \right\}$$

Solve for y .

$$x = 8 - 4(3) = -4$$

Substitute 3 in for y to find the value of x .

The solution is $(-4, 3)$.

Check in both original equations:

$$\begin{array}{ll} 3x + 5y = 3 & x = 8 - 4y \\ 3(-4) + 5(3) = 3 & -4 = 8 - 4(3) \\ -12 + 15 = 3 & -4 = 8 - 12 \\ 3 = 3 & -4 = -4 \end{array}$$

SYSTEMS OF LINEAR EQUATIONS

Example:

Solve the system of linear equations using the substitution method.

$$\begin{cases} 4x + 12y = 4 \\ -5x + y = 11 \end{cases}$$

Since y has a coefficient of 1, it will be easy to solve this second equation for y .

$$\begin{cases} 4x + 12y = 4 \\ y = 5x + 11 \end{cases}$$

$$4x + 12(5x + 11) = 4$$

Substitute $5x + 11$ in for y

$$\begin{cases} 4x + 60x + 132 = 4 \\ 64x = -128 \\ x = -2 \end{cases}$$

Solve for x .

$$\begin{aligned} y &= 5(-2) + 11 \\ y &= 1 \end{aligned}$$

Substitute -2 in for x to find the value of y .

The solution is $(-2, 1)$.

Check in both original equations:

$$\begin{array}{ll} 4x + 12y = 4 & -5x + y = 11 \\ 4(-2) + 12(1) = 4 & -5(-2) + 1 = 11 \\ -8 + 12 = 4 & 10 + 1 = 11 \\ 4 = 4 & 11 = 11 \end{array}$$

SYSTEMS OF LINEAR EQUATIONS

Solving Linear Systems By Elimination (Addition) Method

To solve linear systems of two equations in two variables by elimination:

1. Write the system so that each equation is in standard form.

$$ax + by = c$$

2. Multiply one equation (or both equations if necessary), by a number to obtain additive inverse (opposite) coefficients of one of the variables.
3. Add the resulting equations and solve the new equation in one variable.
4. Substitute the value for that variable into one of the original equations and solve for the other variable.
 - a. If we get $0 = \text{nonzero number}$ or $\# = \text{different } \#$ (contradiction), the system is inconsistent; there are no solutions, the equations are independent, and the lines are parallel.
 - b. If we get $0 = 0$ or $\# = \text{same } \#$ (identity), the system is consistent; there are infinitely many solutions, the equations are dependent and the lines are the same (coincide).

Example:

Solve the system of linear equations using the elimination (addition) method.

$$\begin{cases} x + y = 9 \\ 2x - y = -3 \end{cases}$$

$$3x = 6$$

$$x = 2$$

The coefficients of y are opposites (additive inverses). Add these equations.

Solve for x .

$$x + y = 9$$

$$2 + y = 9$$

$$y = 7$$

Substitute 2 in for x and solve for y .

The solution is (2,7).

Check in both original equations:

$$x + y = 9$$

$$2 + 7 = 9$$

$$9 = 9$$

$$2x - y = -3$$

$$2(2) - 7 = -3$$

$$4 - 7 = -3$$

$$-3 = -3$$

SYSTEMS OF LINEAR EQUATIONS

Example:

Solve this system of linear equations using the elimination (addition) method.

$$\begin{cases} 3x + 2y = 11 \\ 2x - 8y = -2 \end{cases}$$

Multiplying the first equation by 4 yields $8y$. This will result in opposite coefficients (additive inverses) of y .

$$\begin{cases} 12x + 8y = 44 \\ \underline{2x - 8y = -2} \\ 14x = 42 \\ x = 3 \end{cases}$$

Add these equations.

Solve for x .

$$\begin{aligned} 3(3) + 2y &= 11 \\ 9 + 2y &= 11 \\ 2y &= 2 \\ y &= 1 \end{aligned}$$

Substitute 3 in for x and solve for y .

The solution is (3,1).

Check in both original equations:

$$\begin{array}{ll} 3x + 2y = 11 & 2x - 8y = -2 \\ 3(3) + 2(1) = 11 & 2(3) - 8(1) = -2 \\ 9 + 2 = 11 & 6 - 8 = -2 \\ 11 = 11 & -2 = -2 \end{array}$$

SYSTEMS OF LINEAR EQUATIONS

Problem

Solve this System of Linear Equations using all three methods:

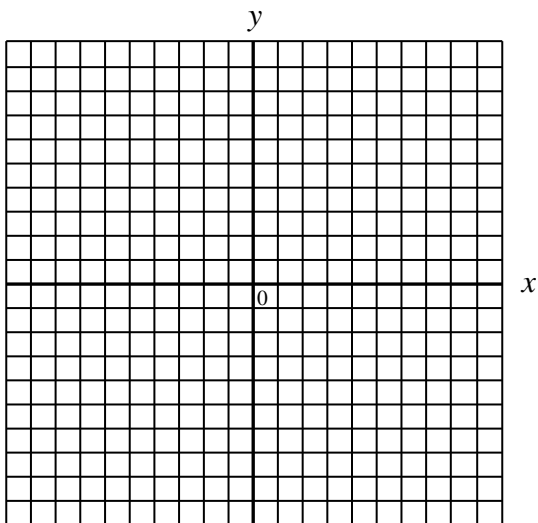
Graphing Method

Substitution Method

Elimination (Addition) Method

$$\begin{cases} x + y = -1 \\ x - y = 1 \end{cases}$$

Graphing Method:



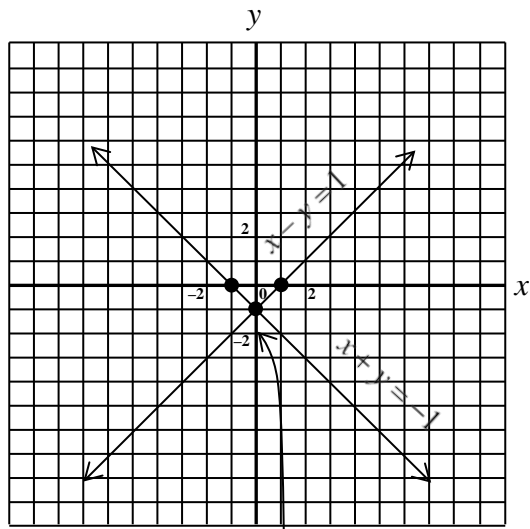
Substitution Method:

Elimination (Addition) Method:

Answer is on the following page.

SYSTEMS OF LINEAR EQUATIONS

Answer:



Solution: $(0, -1)$

SYSTEMS OF LINEAR EQUATIONS

Problems

Solve these systems of linear equations using either the substitution or elimination (addition) method.

1. $x + y = 11$
 $x - y = 1$

$x = 6$ and $y = 5$
 $(6, 5)$

2. $2x + y = 9$
 $3x - y = 11$

$x = 4$ and $y = 1$
 $(4, 1)$

3. $2x + y = 5$
 $5x - 2y = 8$

$x = 2$ and $y = 1$
 $(2, 1)$

4. $3x + 4y = 7$
 $8x - y = 7$

$x = 1$ and $y = 1$
 $(1, 1)$

5. $2x + 3y = 2$
 $10x - 6y = 3$

$x = \frac{1}{2}$ and $y = \frac{1}{3}$
 $\left(\frac{1}{2}, \frac{1}{3}\right)$

6. $2x + 7y = 23$
 $x - 4y = -11$

$x = 1$ and $y = 3$
 $(1, 3)$

7. $x + y = 3$
 $2x + 3y = 8$

$x = 1$ and $y = 2$
 $(1, 2)$

8. $5x + y = 5$
 $2x + 3y = 2$

$x = 1$ and $y = 0$
 $(1, 0)$

9. $4x - 4y = 8$
 $x - y = 1$

Inconsistent system with no solution.

10. $2x - y = 4$
 $6x - 3y = 12$

Dependent system with infinitely many solutions.

APPLICATION OF A SYSTEM OF LINEAR EQUATIONS

The following steps are helpful when solving problems involving two unknown quantities.

1. **Analyze the problem** by reading it carefully to understand the given facts. Often a diagram or table will help you visualize the facts of the problem.
2. **Define variables** to represent the two unknown quantities.
3. Translate the words of the problem to **form two equations** involving each of the two variables.
4. **Solve the system** of equations using graphing, substitution, or elimination (addition) method.
5. **State the conclusion.**
6. **Check the results** in the words of the problem.

Example:

Determine the cost of a quart of pineapple and a container of frozen raspberries for the punch Diane is making. Three (3) quarts of pineapple juice and 4 containers of raspberries will cost \$10. Five (5) quarts of pineapple juice and 2 containers of raspberries will cost \$12.

Let: p = cost of a quart of pineapple juice
 r = cost of a container of raspberries

Define the variables

$$\begin{cases} 3p + 4r = 10 \\ 5p + 2r = 12 \end{cases}$$

Solve the system for “ p ” using elimination (addition) method

$$\begin{array}{r} \begin{cases} 3p + 4r = 10 \\ -10p - 4r = -24 \end{cases} \\ \hline -7p = -14 \\ p = 2 \end{array}$$

$$\begin{array}{r} 3(2) + 4r = 10 \\ r = 1 \end{array}$$

Substitute 2 in for p and solve for r

Answer: A quart of pineapples costs \$2 and a container of frozen raspberries costs \$1.

Check:

Using \$2 as the cost for a quart of pineapple juice, 3 quarts costs \$6. Using \$1 as the cost of a container of raspberries, 4 containers cost \$4. \$6 for pineapple juice and \$4 for raspberries is \$10 total cost. Do the same technique for checking the \$12 cost.

Check the results in the words of the problem.

APPLICATION OF A SYSTEM OF LINEAR EQUATIONS

Problem

Use the steps for solving a system of linear equations.

People have begun purchasing tickets to a production of a musical at a regional theatre. A purchase of 9 adult tickets and 7 tickets for the children costs \$116. Another purchase of 5 adult tickets and 8 tickets for the children costs \$85. Find the cost of an adult ticket and a child's ticket.

Answer

A child's ticket costs \$5
and an adult ticket costs \$9.

SOLVING LINEAR INEQUALITIES IN TWO VARIABLES

Graphing Linear Inequalities In Two Variables

1. Replace the inequality symbol with an equal symbol = and graph the boundary line of the region. If the original inequality allows the possibility of equality (the symbol is either \leq or \geq), draw the boundary line as a solid line. If equality is not allowed ($<$ or $>$), draw the boundary line as a dashed line.

2. Pick a test point that is on one side of the boundary line. (Use the origin if possible; it is easier). Replace x and y in the inequality with the coordinates of that point. If a true statement results, shade the side (half-plane) that contains that point. If a false statement results, shade the other side (other half-plane).

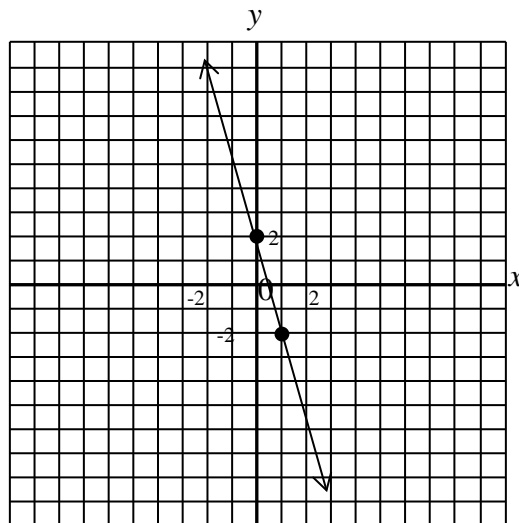
Example:

Solve $4x + y \geq 2$

Step 1: $4x + y = 2$

$$y = -4x + 2$$

\geq sign, draw the solid boundary line



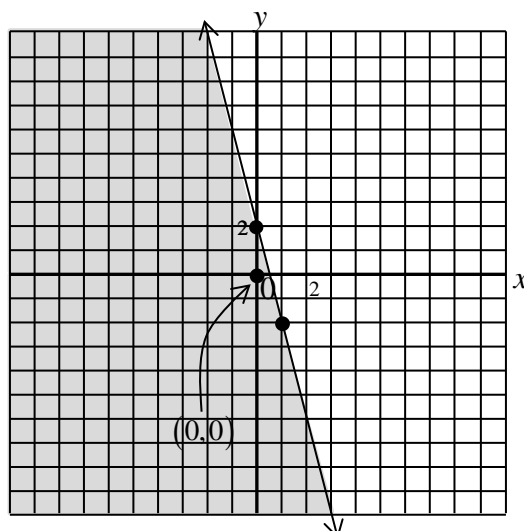
Step 2: pick (0,0)

$$4(0) + 0 \geq 2$$

$$4 \geq 2 \text{ is true}$$

Pick test point.

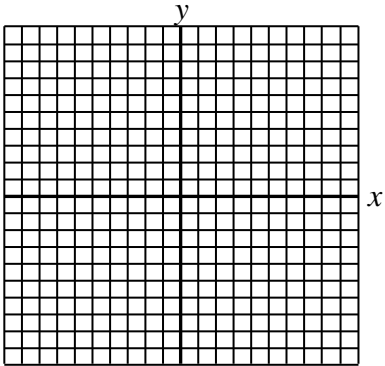
Shade the side that contains (0,0).



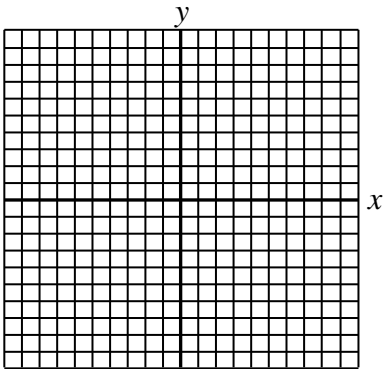
SOLVING LINEAR INEQUALITIES IN TWO VARIABLES

Problems

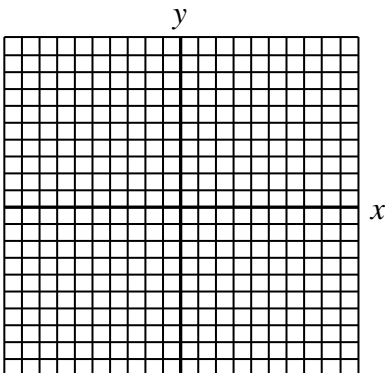
1. Solve $y \leq \frac{1}{2}x - 1$



2. Solve $2y + 3x < 6$

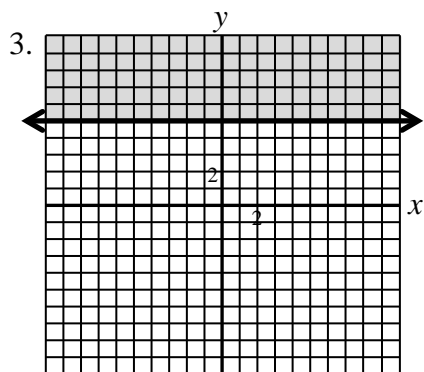
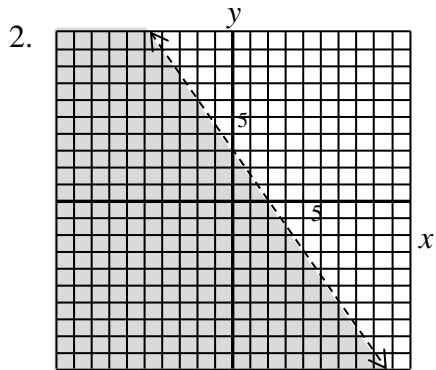
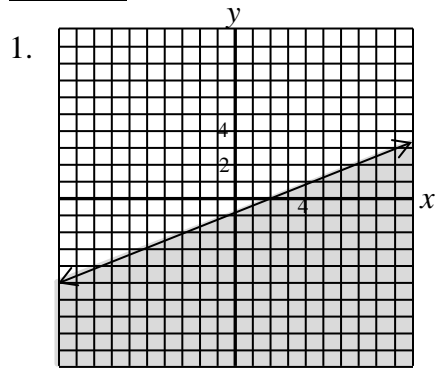


3. Solve $y \geq 5$



SOLVING LINEAR INEQUALITIES IN TWO VARIABLES

Answers



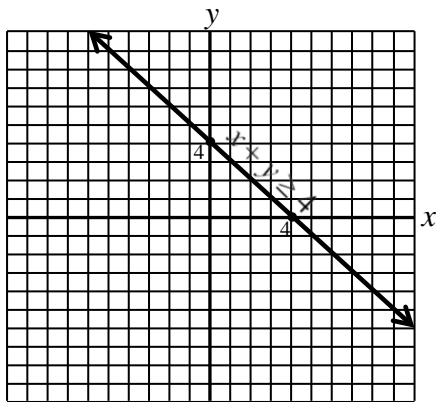
SOLVING SYSTEMS OF LINEAR INEQUALITIES

1. Graph each inequality on the same rectangular coordinate system.
2. Use shading to highlight the intersection of the graphs (the region where the graphs overlap). The points in this region are the solutions of the system.
3. As an informal check, pick a point from the region where the graphs intersect and verify that its coordinates satisfy each inequality of the original system.

Example: Solve this system of inequalities.
$$\begin{cases} x + y \geq 4 \\ 4x - 3y > 9 \end{cases}$$

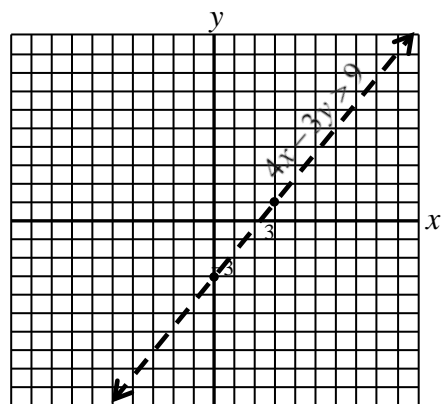
Look at the graph of each inequality separately.

$$x + y \geq 4$$



\geq sign, draw solid boundary line.
Use $(0, 0)$ as a test point.
 $0 \geq 4$ is false.
Shade the half-plane that does not include $(0, 0)$.

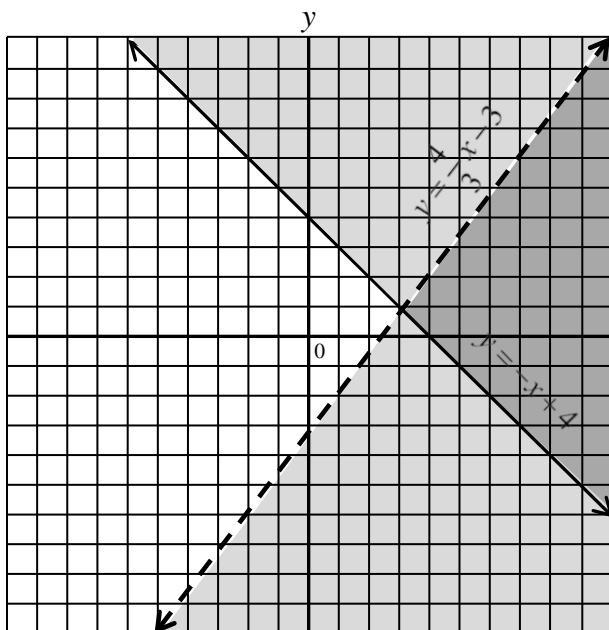
$$4x - 3y > 9$$



$>$ sign, draw dashed boundary line.
Use $(0, 0)$ as a test point.
 $0 > 9$ is false.
Shade the half-plane that does not include $(0, 0)$.

SOLVING SYSTEMS OF LINEAR INEQUALITIES

$$\begin{cases} x + y \geq 4 \\ 4x - 3y > 9 \end{cases}$$

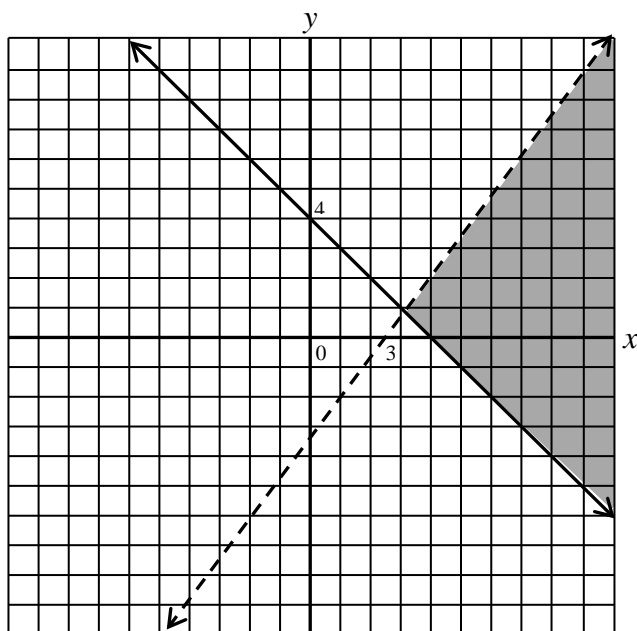


Graph each inequality on the same coordinate system.

Notice the region where the graphs overlap.

The points in this region are the solutions of the system.

Solution of the System:



SOLVING SYSTEMS OF LINEAR INEQUALITIES

Informal Check:

The point (5,2) is in the region of overlap. Substitute (5,2) in to both inequalities.

$$\frac{x + y \geq 4}{5 + 2 \quad | \quad 4}$$

$$7 \geq 4 \text{ is true}$$

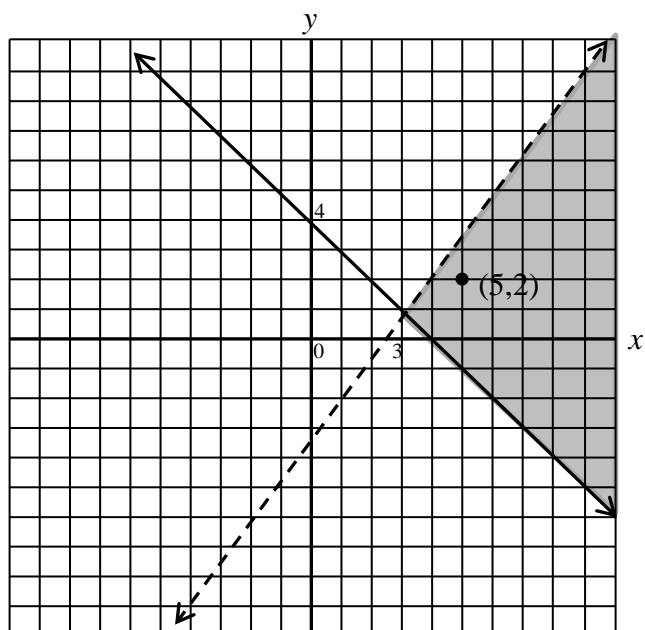
$$\frac{4x - 3y > 9}{4(5) - 3(2) \quad | \quad 9}$$

$$20 - 6 \quad | \quad 9$$

$$14 > 9 \text{ is true}$$

$$14 > 9 \text{ is true}$$

(5,2) is one of the solutions.

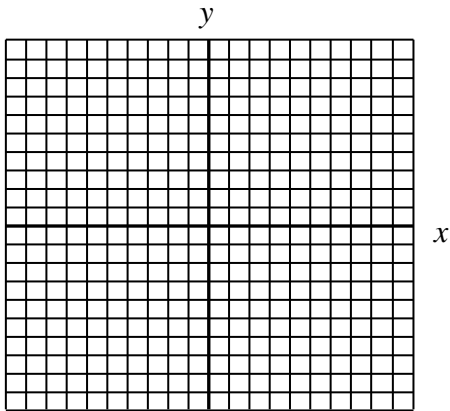


SOLVING SYSTEMS OF LINEAR INEQUALITIES

Problems

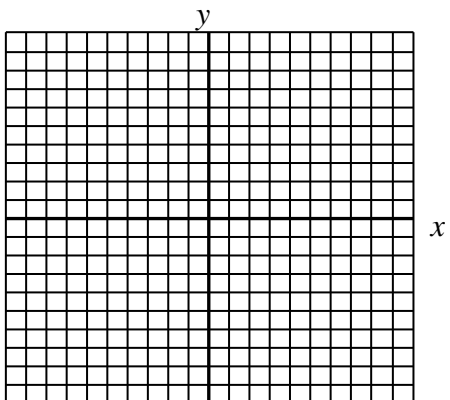
1. Solve this system of linear inequalities.

$$\begin{cases} y \geq -\frac{4}{3}x + 2 \\ y > \frac{5}{6}x + 1 \end{cases}$$



2. Solve this system of linear inequalities.

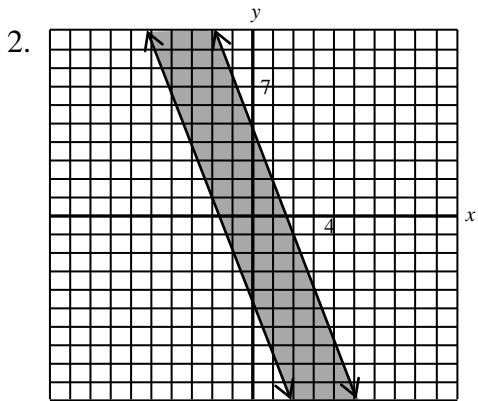
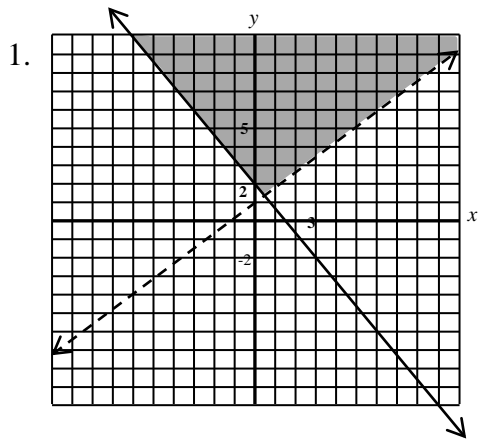
$$\begin{cases} 3x + y \geq -5 \\ -3x - y \geq -5 \end{cases}$$



Answers on the next page

SOLVING SYSTEMS OF LINEAR INEQUALITIES

Answers



SOLVING ABSOLUTE VALUE EQUATIONS

Absolute Value:

The absolute value of a number is its distance from 0 on the number line

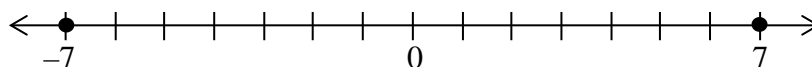
For any positive number k and any algebraic expression X :

To solve $|X|=k$, solve the equivalent compound equation

$$X = k \quad \text{or} \quad X = -k$$

Example: $|x|=7$

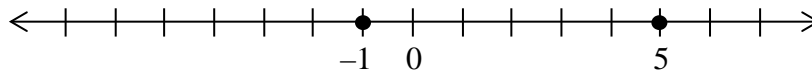
Solution: $x = 7$ or $x = -7$



Example: $|x-2|=3$

$$x-2=3 \quad \text{or} \quad x-2=-3$$

Solution: $x = 5$ or $x = -1$



$$\begin{aligned} \text{Check:} \quad x &= 5 \\ |x-2| &= 3 \\ |5-2| &= 3 \\ |3| &= 3 \\ 3 &= 3 \end{aligned}$$

$$\begin{aligned} \text{Check:} \quad x &= -1 \\ |x-2| &= 3 \\ |-1-2| &= 3 \\ |-3| &= 3 \\ 3 &= 3 \end{aligned}$$

SOLVING ABSOLUTE VALUE EQUATIONS

Example: Solve the following: $\left| \frac{4x-64}{4} \right| = 32$

Solve the equation by rewriting it as two separate equations.

$$\frac{4x-64}{4} = 32 \quad \text{or} \quad \frac{4x-64}{4} = -32$$

When $|X| = k$, then $X = k$
and $X = -k$

Solve each equation for x .

$$4x - 64 = 128$$

$$4x - 64 = -128$$

Multiply both sides by 4.

Solutions:

$$4x = 192$$

$$4x = -64$$

Add 64 to both sides.

$$\mathbf{x = 48} \quad \text{or}$$

$$\mathbf{x = -16}$$

Divide both sides by 4.

Check:

Check $x = 48$

Check $x = -16$

$$\left| \frac{4(48) - 64}{4} \right| = 32$$

$$\left| \frac{4(-16) - 64}{4} \right| = 32$$

$$\left| \frac{192 - 64}{4} \right| = 32$$

$$\left| \frac{-64 - 64}{4} \right| = 32$$

$$\left| \frac{128}{4} \right| = 32$$

$$\left| \frac{-128}{4} \right| = 32$$

$$|32| = 32$$

$$|-32| = 32$$

$$32 = 32$$

$$32 = 32$$

The two solutions check.

SOLVING ABSOLUTE VALUE EQUATIONS

Example: Solve the following: $|5x - 7| = |4(x + 1)|$

Solve the equation by rewriting it as two separate equations.

$$5x - 7 = 4(x + 1) \quad \text{or} \quad 5x - 7 = -[4(x + 1)]$$

When $|X| = k$, then $X = k$ and $X = -k$

Solve each equation for x .

$$5x - 7 = 4x + 4$$

$$x - 7 = 4$$

$$x = 11$$

$$5x - 7 = -[4x + 4]$$

$$5x - 7 = -4x - 4$$

$$9x - 7 = -4$$

$$9x = 3$$

Use the Distributive Property.

Solutions:

$$x = 11$$

or

$$x = \frac{1}{3}$$

Check:

Check $x = 11$

$$|5(11) - 7| = |4(11 + 1)|$$

$$|55 - 7| = |4(12)|$$

$$|48| = |48|$$

$$48 = 48$$

Check $x = \frac{1}{3}$

$$\left| 5\left(\frac{1}{3}\right) - 7 \right| = \left| 4\left(\frac{1}{3} + 1\right) \right|$$

$$\left| \frac{5}{3} - 7 \right| = \left| 4\left(\frac{4}{3}\right) \right|$$

$$\left| \frac{-16}{3} \right| = \left| \frac{16}{3} \right|$$

$$\frac{16}{3} = \frac{16}{3}$$

The two solutions check.

SOLVING ABSOLUTE VALUE EQUATIONS

Problems

1. $|3x - 2| = 5$

2. $|10x - x| = -40$

3. $\left|\frac{2}{3}x + 3\right| + 4 = 10$

4. $3\left|\frac{1}{2}x - 5\right| - 4 = -4$

5. $|5x + 3| = |3x + 25|$

6. $|2x + 1| = |3(x + 1)|$

Answers

$$x = \frac{7}{3} \quad \text{or} \quad x = -1$$

No solution. You can't solve an absolute value equation when the absolute value is equal to a negative quantity.

$$x = \frac{9}{2} \quad \text{or} \quad x = -\frac{27}{2}$$

$$x = 10$$

$$x = 11 \quad \text{or} \quad x = -\frac{7}{2}$$

$$x = -2 \quad \text{or} \quad x = -\frac{4}{5}$$

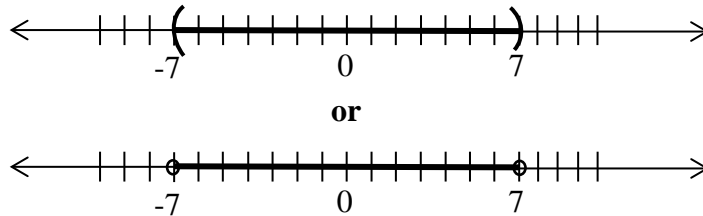
SOLVING ABSOLUTE VALUE INEQUALITIES

To solve inequalities with absolute value signs, there are two cases:

For any positive number k and any algebraic expression X
Case 1: To solve $|X| < k$, solve the equivalent compound inequality $-k < X < k$.
To solve $|X| \leq k$, solve the equivalent compound inequality $-k \leq X \leq k$.

Example: Solve: $|x| < 7$

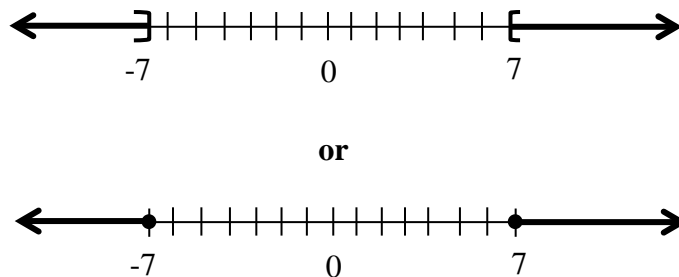
Solution: $-7 < x < 7$ **Interval Notation:** $(-7,7)$



For any positive number k and any algebraic expression X
Case 2: To solve $|X| > k$, solve the equivalent compound inequality $X < -k$ or $X > k$.
To solve $|X| \geq k$, solve the equivalent compound inequality $X \leq -k$ or $X \geq k$.

Example: Solve: $|x| \geq 7$

Solution: $x \leq -7$ or $x \geq 7$ **Interval Notation:** $(-\infty, -7] \cup [7, \infty)$



SOLVING ABSOLUTE VALUE INEQUALITIES

Example:

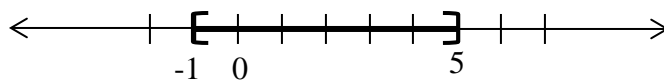
Solve: $|x-2| \leq 3$
 $-3 \leq x-2 \leq 3$
 $-1 \leq x \leq 5$

Case 1

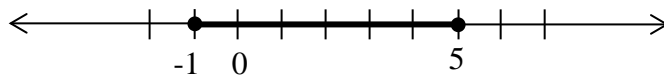
Equivalent compound inequality
 Addition Property of Equality

Solution: $-1 \leq x \leq 5$

Interval Notation: $[-1,5]$



or



Example:

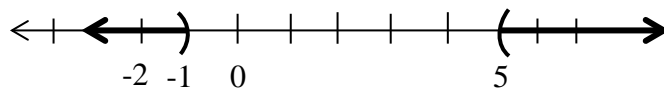
Solve: $|x-2| > 3$
 $x-2 < -3$ or $x-2 > 3$
 $x < -1$ $x > 5$

Case 2

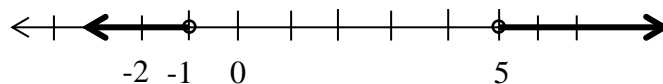
Equivalent inequalities

Solution: $x < -1$ or $x > 5$

Interval Notation: $(-\infty, -1) \cup (5, \infty)$



or



SOLVING ABSOLUTE VALUE INEQUALITIES

Problems

1. $|x-5| < 3$

2. $|x-7| > 10$

3. $|5-3x| \leq 14$

4. $|2x-3| < 9$

5. $\left| \frac{3-x}{5} \right| \geq 6$

6. $|3x-8|-4 \geq 0$

Answers

$$2 < x < 8 \\ (2, 8)$$

$$x < -3 \text{ or } x > 17 \\ (-\infty, -3) \cup (17, \infty)$$

$$-3 \leq x \leq \frac{19}{3} \\ \left[-3, \frac{19}{3} \right]$$

$$-3 < x < 6 \\ (-3, 6)$$

$$x \leq -27 \text{ or } x \geq 33 \\ (-\infty, -27] \cup [33, \infty)$$

$$x \leq \frac{4}{3} \text{ or } x \geq 4 \\ \left(-\infty, \frac{4}{3} \right] \cup [4, \infty)$$

VOCABULARY USED IN APPLICATION PROBLEMS

Addition

	English	Algebra
add		
sum	The sum of a number and 4.	$x + 4$
total	Seven more than a number.	$x + 7$
plus	Six increased by a number.	$6 + x$
in all	A number added to 8.	$8 + x$
more than	A number plus 4.	$x + 4$
together		
increased by		
all together		
combined		

Subtraction

	English	Algebra
subtracted from		
difference	The difference of a number and 3.	$x - 3$
take away	The difference of 3 and a number.	$3 - x$
less than	Five less than a number.	$x - 5$
minus	A number decreased by 3.	$x - 3$
remain	A number subtracted from 8.	$8 - x$
decreased by	Eight subtracted from a number.	$x - 8$
have left	Two minus a number.	$2 - x$
are left		
more		
fewer		

Be careful with subtraction. The order is important. Three less than a number is $x - 3$.

VOCABULARY USED IN APPLICATION PROBLEMS

Multiplication

product of
multiplied by
times
of

English

Algebra

The **product** of 3 and a number.

$3x$

Three-fourths **of** a number.

$\frac{3}{4}x$

Four **times** a number.

$4x$

A number **multiplied** by 6.

$6x$

Double a number.

$2x$

Twice a number.

$2x$

Division

divided by
quotient of
separated into equal
parts
shared equally

English

Algebra

The **quotient** of a number and 3.

$x \div 3$ or $\frac{x}{3}$

The **quotient** of 3 and a number.

$3 \div x$ or $\frac{3}{x}$

A number **divided by** 6.

$x \div 6$ or $\frac{x}{6}$

Six **divided by** a number.

$6 \div x$ or $\frac{6}{x}$

Be careful with division. The order is important. A number divided by 6 is $\frac{x}{6}$

SOLVING APPLICATION PROBLEMS

Strategy for solving word problems:

1. **Analyze the problem:** Read the problem carefully.
2. **Visualize the facts of the problem (if needed):** Use diagrams and/or tables.
3. **Define the variable/s:** Identify the unknown quantity (or quantities) and label them, i. e., let $x = \text{something}$.
4. **Write an equation:** Use the defined variable/s.
5. **Solve the equation:** Make sure you have **answered the question** that was asked.
6. **Check your answer(s):** Use the original words of the problem.

Examples:

1. The sum of three times a number and 11 is -13 . Find the number.

Let $x = \text{the number}$

Define the variable.

$$3x + 11 = -13$$

Write an equation..

$$3x + 11 - 11 = -13 - 11$$

$$3x = -24$$

$$x = -8$$

Solve the equation.

Solution: The number is -8 .

$$\text{Check: } 3(-8) = -24$$

$$-24 + 11 = -13$$

2. Together, a lot and a house cost $\$40,000$. The house costs seven times more than the lot. How much does the lot cost? The house?

Let $x = \text{the cost of the lot}$

$7x = \text{the cost of the house}$

Define the variable.

$$x + 7x = 40,000$$

Write an equation

$$8x = 40,000$$

$$x = 5,000$$

$$7x = 7 \cdot 5000 = 35,000$$

Solve the equation.

Solution: $\left\{ \begin{array}{l} \text{The lot costs } \$5,000. \\ \text{The house costs } \$35,000. \end{array} \right.$

$$\text{Check: } 5,000 + 35,000 = 40,000$$

$$7 \cdot 5000 = 35,000$$

SOLVING APPLICATION PROBLEMS

Problems:

1. Five plus three more than a number is nineteen. What is the number?
2. When 18 is subtracted from six times a certain number, the result is 96. What is the number?
3. If you double a number and then add 85, you get three-fourths of the original number. What is the original number?
4. A 180-m rope is cut into three pieces. The second piece is twice as long as the first. The third piece is three times as long as the second. How long is each piece of rope?
5. Donna and Melissa purchased rollerblades for a total of \$107. Donna paid \$17 more for her rollerblades than Melissa did. What did Melissa pay?
6. A student pays \$278 for a calculator and a keyboard. If the calculator costs \$64 less than the keyboard, how much did each cost?

SOLVING APPLICATION PROBLEMS

Answers

1. $5 + (x + 3) = 19$
 $x = 11$

The number is 11.

2. $6x - 18 = 96$
 $x = 19$

The number is 19.

3. $2x + 85 = \frac{3}{4}x$
 $x = -68$

The number is -68 .

4. $x + 2x + 3(2x) = 180$
 $x = 20$

The lengths are 20 m, 40 m, and 120 m.

5. $x + x + 17 = 107$
 $x = 45$

Melissa paid \$45.

6. $x + (x - 64) = 278$
 $x = 171$

The keyboard costs \$171, and the calculator costs \$107.

SOLVING APPLICATION PROBLEMS

Problems

Solve the following word problems using one variable or two variables in a system.

Integer Problems

1. The sum of three consecutive integers is 144. Find the integers.
2. The sum of three consecutive even integers is 84. Find the integers.
3. The sum of three consecutive odd integers is 111. Find the integers.

Perimeter Problems

4. The length of a rectangle is seven more than the width. The perimeter is 34 inches. Find the length and width.
5. The length of a rectangle is 3 inches less than twice the width. The perimeter is 18 inches. Find the length and width.
6. The length of one side of a triangle is 4 inches more than the shortest side. The longest side is two inches more than twice the length of the shortest side. The perimeter is 26 inches. Find the length of all three sides.

SOLVING APPLICATION PROBLEMS

Mixture Problems

7. For Valentine's Day, Candy, a candy store owner, wants a mixture of candy hearts and foil-wrapped chocolates. If she has 10 pounds of candy hearts, which sell for \$2 per pound, how many pounds of the chocolates, which sell for \$6 per pound, should be mixed to get a mixture selling at \$5 per pound?
8. The same candy store owner has 24 pounds of chocolate creams which sell for \$12 per pound. How many pounds of chocolate caramel nut clusters, which sell for \$9 per pound, should he mix to get a mixture selling at \$10 per pound?
9. Sharon wants to make 100 pounds of holiday mix for her baskets. She purchases cashews at \$8.75 per pound and walnuts at \$3.75 per pound. She feels she can afford a mixture which costs \$6.35 per pound. How much of each type of nut should she purchase to make the mix?

Distance Problems

10. Two cars leave Chicago at 11 a.m. headed in opposite directions. At 2 p.m., the two cars are 375 miles apart. If one car is traveling 5 mph faster than the other, what are their speeds?
11. A plane leaves Chicago headed due west at 10 a.m. At 11 a.m., another plane leaves Chicago headed due east. At 1 p.m., the two planes are 2950 miles apart. If the first plane is flying 100 mph slower than the second plane, find their rates.
12. A car leaves Milwaukee at noon headed north. Five hours later it arrives at its destination. A second car traveling south at a rate 10 mph slower leaves Milwaukee at 3:00 and arrives at its destination two hours later. When they arrive at their destination, they are 400 miles apart. How fast is each of the cars traveling?
13. A bicyclist can ride 24 miles with the wind in 2 hours. Against the wind, the return trip takes him 3 hours. Find the speed of the wind. (Hint: Solve using systems of equations.)

SOLVING APPLICATION PROBLEMS

Investment Problems

14. A couple wants to invest \$12,000 in two retirement accounts, one earning 6% and the other 9%. How much should be invested in each account for them to earn an annual interest of \$945?
15. An investor has put \$4,500 in a credit union account earning 4% annual interest. How much should he invest in an account which pays 10% annual interest to receive total annual interest of \$1,000 from the two accounts.
16. Three accounts generate a total annual interest of \$1,249.50. The investor deposited an equal amount of money in each account. The accounts paid an annual rate of return of 7%, 8%, and 10.5%. How much was invested in each account?

Number-Value Problems

17. The admission prices for a movie theater in Crystal Lake are \$9 for adults, \$8 for seniors, and \$5 for children. A family purchased twice as many children's tickets as adults and the same number of senior tickets as adults. The total cost of the tickets was \$54. How many of each type of ticket was purchased?
18. Marie has \$2.20 worth of quarters, dimes, and nickels. She has 3 times as many nickels as quarters and 3 fewer dimes than quarters. How many of each type of coin does she have?
19. Deb went shopping at the school bookstore and purchased \$53 of computer items. The CD's cost \$2 each, the DVD's cost \$3 each, and the flash drives were \$15 each. The total cost was \$53. He purchased one more DVD than CD's and half as many flash drives as CD's. How many of each did he purchase?

SOLVING APPLICATION PROBLEMS

Answers

1. The 3 consecutive integers are 47, 48, and 49.
2. The 3 consecutive even integers are 26, 28, and 30.
3. The 3 consecutive odd integers are 35, 37, and 39.
4. The length is 12 inches, and the width is 5 inches.
5. The length is 5 inches, and the width is 4 inches.
6. The lengths are 5 inches, 9 inches, and 12 inches.
7. The store owner should add 30 pounds of the chocolates worth \$6 per pound.
8. The store owner should mix 48 pounds of chocolate caramel nut clusters worth \$9 per pound.
9. 52 pounds of cashews and 48 pounds of walnuts.
10. One car is traveling 60 mph; the other is traveling 65 mph.
11. The first plane is traveling 550 mph. The second plane is traveling 650 mph.
12. One car is traveling 60 mph; the other is traveling 50 mph.
13. The wind speed is 2 miles per hour.
14. \$4,500 invested at 6% and \$7,500 at 9%.
15. \$8,200 at 10%.
16. \$4,900 in each account.
17. 2 adult tickets, 4 children's, and 2 senior tickets.
18. 5 quarters, 2 dimes, and 15 nickels.
19. 4 CD's, 5 DVD's, and 2 flash drives.

COMPREHENSIVE REVIEW OF ELEMENTARY ALGEBRA

Problems:

1. Evaluate the following:

a. $-3^2 + |4^2 - 5^2|$

b. $(4-5)^{20}$

c. $\frac{-3 - (-7)}{2^2 - 3}$

d. $12 - 2[1 - (-8 + 2)]$

2. If $x = 2$ and $y = -4$, evaluate:

a. $|x| - xy$

b. $\frac{x^2 - y^2}{3x + y}$

3. Solve the following equations for x :

a. $3(x-5) + 2 = 2x$

b. $\frac{x-5}{3} - 5 = 7$

c. $\frac{2}{5}x + 1 = \frac{1}{3} + x$

d. $\frac{-5}{8}x = 15$

COMPREHENSIVE REVIEW OF ELEMENTARY ALGEBRA

4. Solve the following for the indicated variable:

a. Solve: $P = 2l + 2w$ for w .

b. Solve: $A = \frac{1}{2}bh$ for h .

c. Solve: $3x + 2y = 5$ for y .

d. Solve: $\frac{a}{2} + \frac{b}{3} = \frac{c}{4}$ for a .

5. Solve the following inequalities. Leave your answers in interval form.

a. $3 - (2x + 4) < -2(x + 3) + x$

b. $-5x + 4 \geq 6$

6. Simplify the following:

a. $(-3x^2y^2)^2$

b. $(2y)^{-4}$

c. $(x^3x^4)^2$

d. $(ab^{-3}c^4)(ab^4c^{-2})$

e. $\frac{a^4b^0}{a^{-3}}$

f. $\left(\frac{4t^{-3}t^4t^{-5}}{3t^2t^{-6}}\right)^3$

COMPREHENSIVE REVIEW OF ELEMENTARY ALGEBRA

7. Perform the indicated operations:

a. $(4c^2 + 3c - 2) + (3c^2 + 4c + 2)$

b. $3x(2x + 3)^2$

c. $(2t + 3s)(3t - s)$

d. $(10x^2 + 11x + 3) \div (5x + 3)$

e. $(5x - 8y) - (-2x + 5y)$

f. $(-3x + y)(x^2 - 8xy + 16y^2)$

g. $(2x - 32) \div 16x$

h. $(x + y)(x - y) + x(x + y)$

8. Graph the following:

a. $4x - 3y = 12$

b. $x = 4$

c. $3x - 8y = 0$

d. $y = -2$

9. Find the x- and y-intercept of the following:

a. $3x - 2y = 5$

b. $2x - 3y = 0$

c. $x = 4$

d. $y = -2$

COMPREHENSIVE REVIEW OF ELEMENTARY ALGEBRA

10. Find the equation of the following lines:

- a. A line passing through (-2,4) and (6, 10)
- b. A line with a slope of $\frac{2}{3}$ passing through (0, 5)
- c. A line through (1, 2) with a slope of $\frac{1}{2}$
- d. A horizontal line through (-3, 5)
- e. A vertical line through (-3, 5)
- f. A line with an undefined slope through (3, -2)

11. Find the slope of the following:

- a. A line through (-3, 7) and (2, -5)
- b. The line $3x - 8y = 4$
- c. The line $y = 7$
- d. The line $x = -4$

12. Solve the following systems:

a.
$$\begin{cases} x + 4y = -2 \\ y = -x - 5 \end{cases}$$

b.
$$\begin{cases} 3x - y = 2 \\ 2x + y = 8 \end{cases}$$

c.
$$\begin{cases} 3x - 5y - 16 = 0 \\ \frac{x}{2} - \frac{5}{6}y = \frac{1}{3} \end{cases}$$

d.
$$\begin{cases} x = -3y + 6 \\ 2x + 6y = 12 \end{cases}$$

e.
$$\begin{cases} x - 4 = y \\ -2y = 4 - 2x \end{cases}$$

COMPREHENSIVE REVIEW OF ELEMENTARY ALGEBRA

13. Solve the following inequalities. Leave your answer in interval notation.
- $3x - 2 < 4$ and $-3x < 3$
 - $-2 \leq 2x - 5 < 6$
 - $-2x + 4 > 8$ or $x - 1 \geq 2$
 - $|3x - 2| < 4$
 - $|2x + 7| \geq 4$
 - $-2|3x - 1| > 6$
 - $|2x - 4| - 3 < 2$
14. Factor.
- $14xyz - 16x^2y^2z$
 - $-r^2 - 14r - 45$
 - $6s^5 - 26s^4 - 20s^3$
 - $9z^2 - 1$
15. The length of a rectangle is one foot longer than twice the width. The perimeter is 20 inches. Find the length and width.
16. The longest side of a triangle is two centimeters longer than twice the shortest side. The medium length side is three centimeters longer than the shortest side. The perimeter is 25 centimeters. Find the lengths of the three sides.
17. One train leaves Chicago at 7 p.m. At 9 p.m., another train leaves Chicago heading in the opposite direction, traveling 10 mph faster. At 11 p.m., the two trains are 530 miles apart. How fast was each train moving?
18. Scotty inherited \$20,000 which he invested in two different accounts. The first account paid out an interest rate of 6%. The second account paid out an interest rate of 3%. After 1 year, Scotty had earned \$750 interest. How much did he invest in each account

COMPREHENSIVE REVIEW OF ELEMENTARY ALGEBRA

Answers

1. a. 0

b. 1

c. 4

d. -2

2. a. 10

b. -6

3. a. 13

b. 41

c. $\frac{10}{9}$

d. -24

4. a. $w = \frac{P}{2} - l$ or $w = \frac{P - 2l}{2}$

b. $h = \frac{2A}{b}$

c. $y = -\frac{3}{2}x + \frac{5}{2}$

d. $a = \frac{1}{2}c - \frac{2}{3}b$ or $a = \frac{3c - 4b}{6}$

5. a. $(5, \infty)$

b. $\left(-\infty, -\frac{2}{5}\right]$

COMPREHENSIVE REVIEW OF ELEMENTARY ALGEBRA

Answers (continued)

6. a. $9x^4y^4$

b. $\frac{1}{16y^4}$

c. x^{14}

d. a^2bc^2

e. a^7

f. $\frac{64}{27}$

7. a. $7c^2 + 7c$

b. $12x^3 + 36x^2 + 27x$

c. $6t^2 + 7st - 3s^2$

d. $2x + 1$

e. $7x - 13y$

f. $-3x^3 + 25x^2y - 56xy^2 + 16y^3$

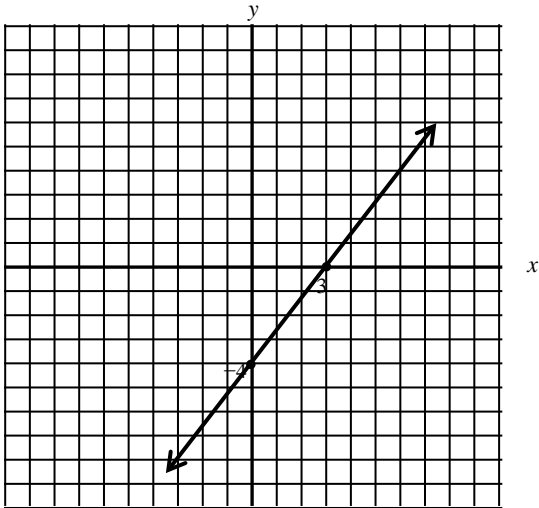
g. $\frac{1}{8} - \frac{2}{x}$

h. $2x^2 + xy - y^2$

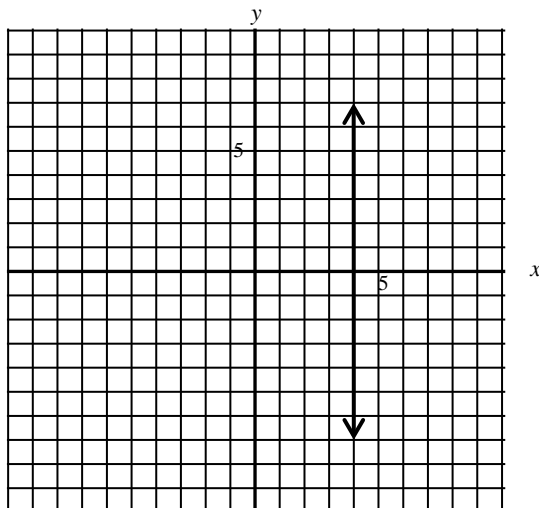
COMPREHENSIVE REVIEW OF ELEMENTARY ALGEBRA

Answers (continued)

8. a. $4x - 3y = 12$



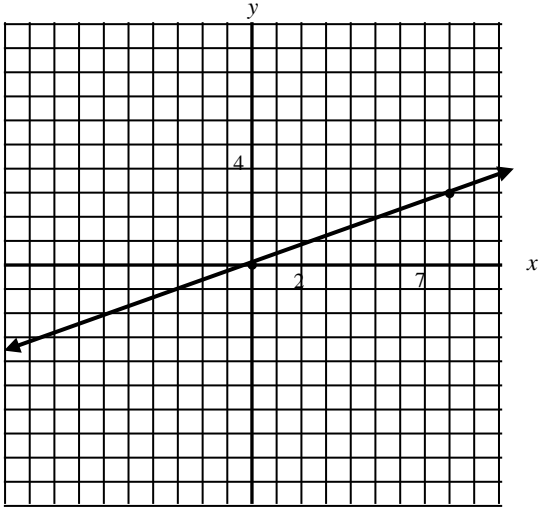
8. b. $x = 4$



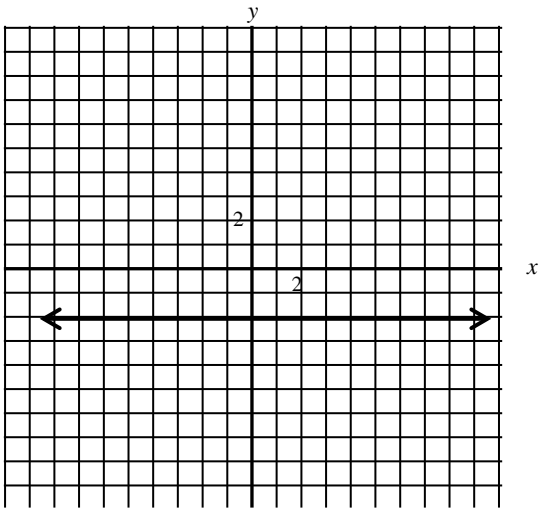
COMPREHENSIVE REVIEW OF ELEMENTARY ALGEBRA

Answers (continued)

8. c. $3x - 8y = 0$



8. d. $y = -2$



COMPREHENSIVE REVIEW OF ELEMENTARY ALGEBRA

Answers (continued)

9. a. x-intercept = $\left(\frac{5}{3}, 0\right)$

y-intercept = $\left(0, -\frac{5}{2}\right)$

b. x-intercept = (0, 0)

y-intercept = (0, 0)

c. x-intercept = (4, 0)

No y-intercept

d. No x-intercept

y-intercept (0, -2)

10. a. $y = \frac{3}{4}x + \frac{11}{2}$

b. $y = \frac{2}{3}x + 5$

c. $y = \frac{1}{2}x + \frac{3}{2}$

d. $y = 5$

e. $x = -3$

f. $x = 3$

11. a. $m = -\frac{12}{5}$

b. $m = \frac{3}{8}$

c. $m = 0$

d. Slope undefined

COMPREHENSIVE REVIEW OF ELEMENTARY ALGEBRA

Answers (continued)

12. a. $(-6, 1)$
b. $(2, 4)$
c. No solution
d. Infinite solutions
e. No solution
13. a. $(-1, 2)$
b. $\left(\frac{3}{2}, \frac{11}{2}\right)$
c. $(-\infty, -2) \cup [3, \infty)$
d. $\left(-\frac{2}{3}, 2\right)$
e. $\left(-\infty, -\frac{11}{2}\right] \cup \left[\frac{3}{2}, \infty\right)$
f. no solution
g. $\left(-\frac{1}{2}, \frac{9}{2}\right)$
14. a. $2xyz(7 - 8xy)$
b. $-(r+9)(r+5)$
c. $2s^3(3s+2)(s-5)$
d. $(3z+1)(3z-1)$
15. The length is 7 feet and the width is 3 feet.
16. Short side is 5 cm, medium side is 8 cm, and the longest side is 12 cm.
17. The train that left at 7:00 p.m., travelled at 85 mph.
The train that left at 9:00 p.m., travelled at 95 mph.
18. \$15,000 at 3% and \$5,000 at 6%.